Depósitos térmicos en serie

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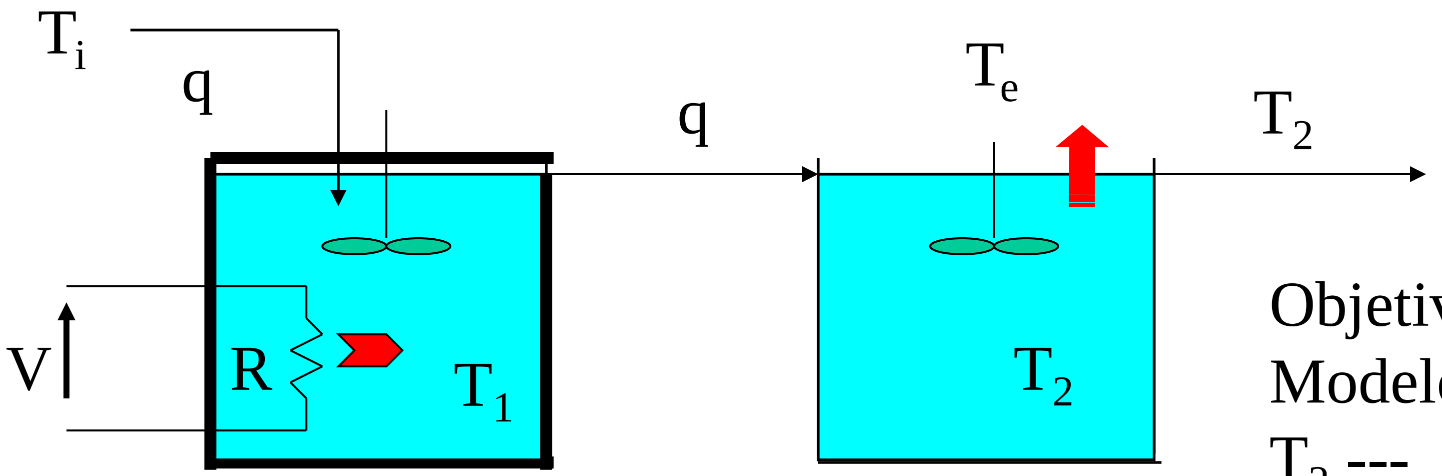
Depósitos térmicos en serie



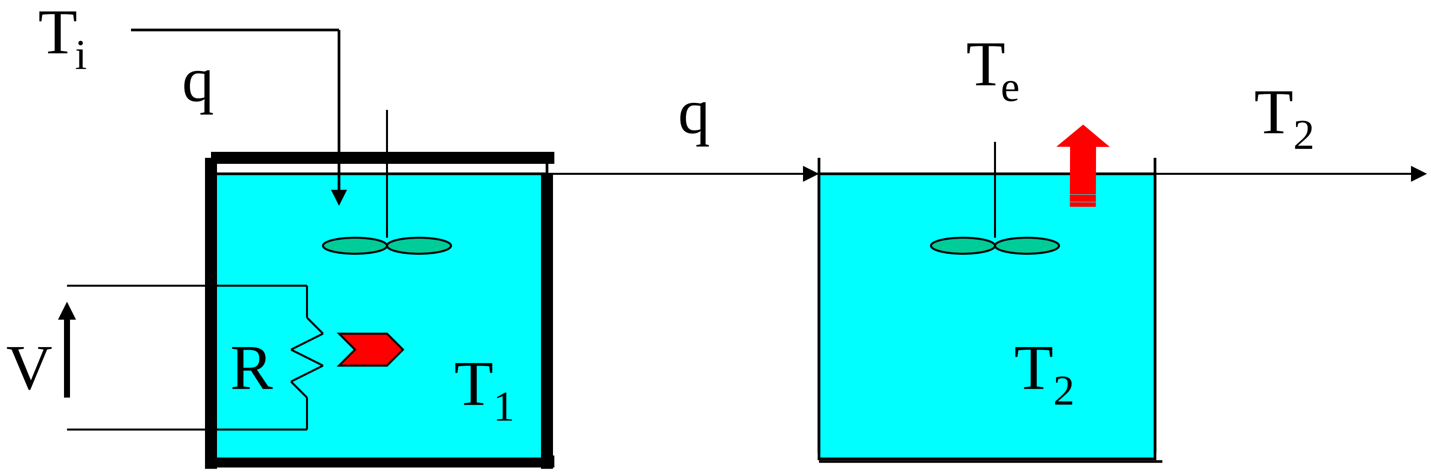
Objetivo:
Modelo entre
T_2 --- V , q

Hipótesis:

T uniforme en el depósito
Aislamiento perfecto 1er dep
densidad constante
caudal por rebose
T_i T_e constantes

T temperatura, V voltaje
m masa en el depósito
H entalpia, c_e calor específico
A sección del depósito
ρ densidad, R resistencia

Depósitos térmicos en serie



$$\frac{d(m_1H_1)}{dt} = q\rho H_i - q\rho H_1 + \frac{V^2}{R}$$

si $\quad H_1 = c_e T_1 \qquad m_1 = A_1 h_1 \rho$

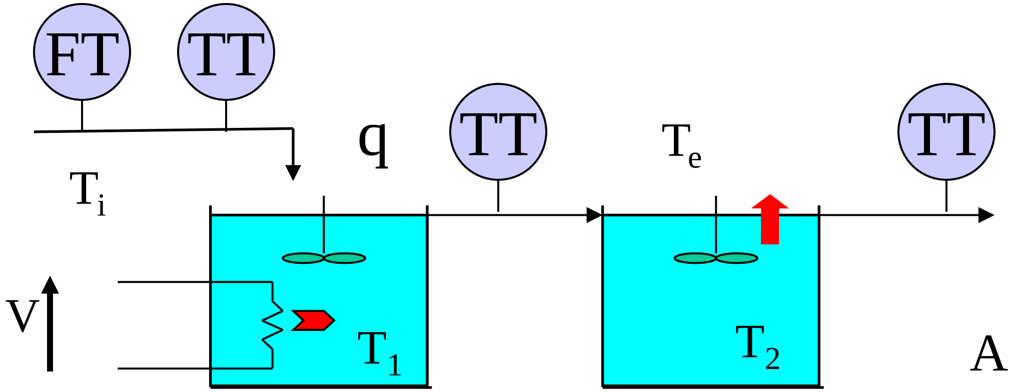
$$A_1 h_1 \frac{dT_1}{dt} = q(T_i - T_1) + \frac{V^2}{\rho c_e R}$$

$$\frac{d(m_2H_2)}{dt} = q\rho H_1 - q\rho H_2 - \alpha(T_2 - T_e)$$

si $\quad H_2 = c_e T_2 \qquad m_2 = A_2 h_2 \rho$

$$A_2 h_2 \frac{dT_2}{dt} = q(T_1 - T_2) - \frac{\alpha}{\rho c_e}(T_2 - T_e)$$

Estimación de parámetros



$$A_1 h_1 \frac{dT_1}{dt} = q(T_i - T_1) + \frac{V^2}{\rho c_e R}$$

$$A_2 h_2 \frac{dT_2}{dt} = q(T_1 - T_2) - \frac{\alpha}{\rho c_e}(T_2 - T_e)$$

En estado estacionario:

$$0 = q(T_i - T_1) + \frac{V^2}{\rho c_e R} \qquad \rho c_e R = \frac{V^2}{q(T_1 - T_i)}$$

$$0 = q(T_1 - T_2) - \frac{\alpha}{\rho c_e}(T_2 - T_e) \Rightarrow \frac{\alpha}{\rho c_e} = \frac{q(T_1 - T_2)}{(T_2 - T_e)}$$

Ah = volumen del depósito

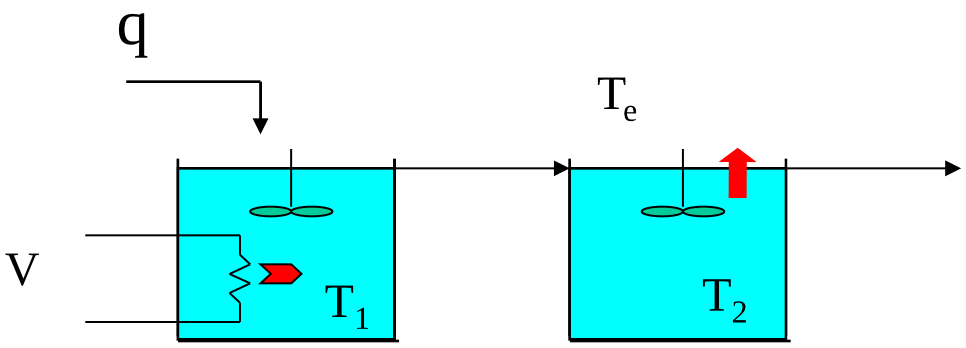
Si T_2 = 80 ºC T_1 = 90 ºC T_i = 30 ºC T_e = 10 ºC q = 5 m^3/h V = 228 volts

$\rho c_e R$ = 173.28

$\alpha/\rho c_e$ = 0.714

Ah = 1 m^3

Linealización



$$A_1 h_1 \frac{dT_1}{dt} = q(T_i - T_1) + \frac{V^2}{\rho c_e R}$$

$$A_2 h_2 \frac{dT_2}{dt} = q(T_1 - T_2) - \frac{\alpha}{\rho c_e}(T_2 - T_e)$$

Linealización 1ª ecuación:

$$A_1 h_1 \frac{d\Delta T_1}{dt} = -q_0 \Delta T_1 + (T_i - T_{01})\Delta q + \frac{2V_0}{\rho c_e R}\Delta V$$

$$\boxed{\frac{A_1 h_1}{q_0}} \frac{d\Delta T_1}{dt} + \Delta T_1 = \boxed{\frac{(T_i - T_{01})}{q_0}} \Delta q + \boxed{\frac{2V_0}{\rho c_e R q_0}} \Delta V$$

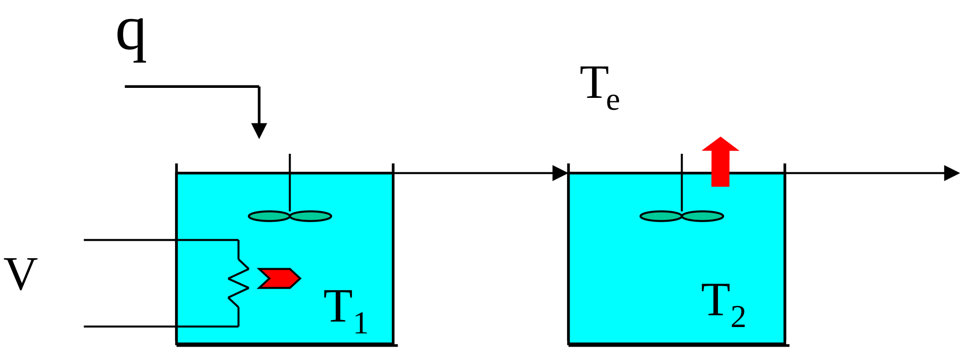
$$\tau_1 \frac{d\Delta T_1}{dt} + \Delta T_1 = K_1 \Delta q + K_2 \Delta V$$

τ_1 = 0.2 horas

K_1 = -12 K_2 = 0.526

τ = volumen/caudal

Linealización



$$A_1 h_1 \frac{dT_1}{dt} = q(T_i - T_1) + \frac{V^2}{\rho c_e R}$$

$$A_2 h_2 \frac{dT_2}{dt} = q(T_1 - T_2) - \frac{\alpha}{\rho c_e}(T_2 - T_e)$$

Linealización
2ª ecuación:

$$A_2 h_2 \frac{d\Delta T_2}{dt} = -(q_0 + \frac{\alpha}{\rho c_e})\Delta T_2 + q_0 \Delta T_1 + (T_{01} - T_{02})\Delta q$$

$$\frac{A_2 h_2}{q_0 + \frac{\alpha}{\rho c_e}} \frac{d\Delta T_2}{dt} + \Delta T_2 = \frac{q_0}{q_0 + \frac{\alpha}{\rho c_e}} \Delta T_1 + \frac{(T_{01} - T_{02})}{q_0 + \frac{\alpha}{\rho c_e}} \Delta q$$

$$\tau_2 \frac{d\Delta T_2}{dt} + \Delta T_2 = K_4 \Delta T_1 + K_3 \Delta q$$

$\tau_2 = 0.175$

$K_4 = 0.785$ $K_3 = 1.75$

Función de Transferencia

Tomando transformadas de Laplace a ambos lados de cada ecuación:

$$\tau_1 \frac{d\Delta T_1}{dt} + \Delta T_1 = K_1 \Delta q + K_2 \Delta V \quad \Rightarrow \tau_1 s \Delta T_1(s) + \Delta T_1(s) = K_1 \Delta q(s) + K_2 \Delta V(s)$$

$$\Delta T_1(s) = \frac{K_1}{\tau_1 s + 1} \Delta q(s) + \frac{K_2}{\tau_1 s + 1} \Delta V(s)$$

$$\tau_2 \frac{d\Delta T_2}{dt} + \Delta T_2 = K_4 \Delta T_1 + K_3 \Delta q \quad \Rightarrow \tau_2 s \Delta T_2(s) + \Delta T_2(s) = K_4 \Delta T_1(s) + K_3 \Delta q(s)$$

$$\Delta T_2(s) = \frac{K_4}{\tau_2 s + 1} \Delta T_1(s) + \frac{K_3}{\tau_2 s + 1} \Delta q(s)$$

Diagrama de bloques (1)

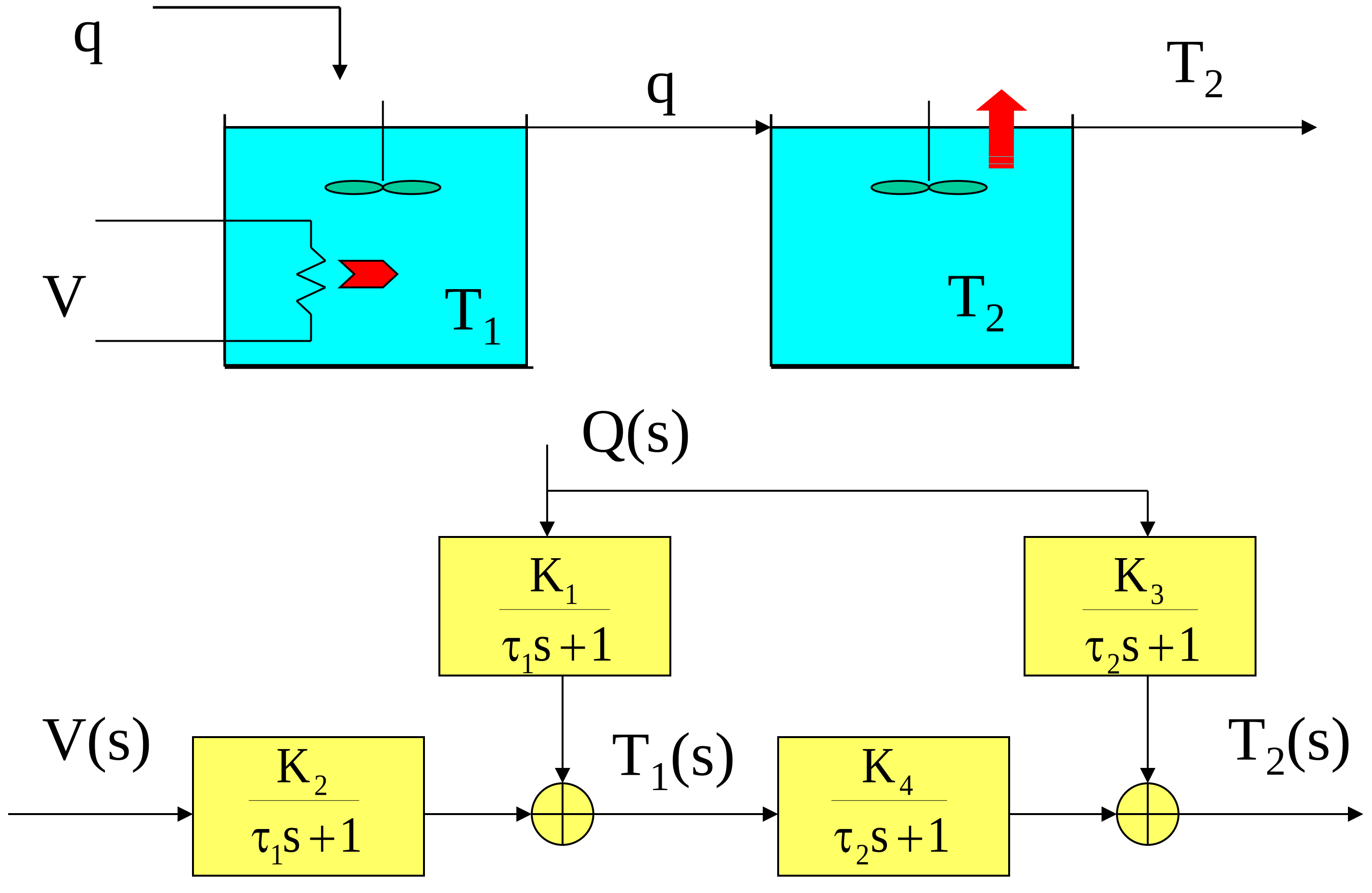
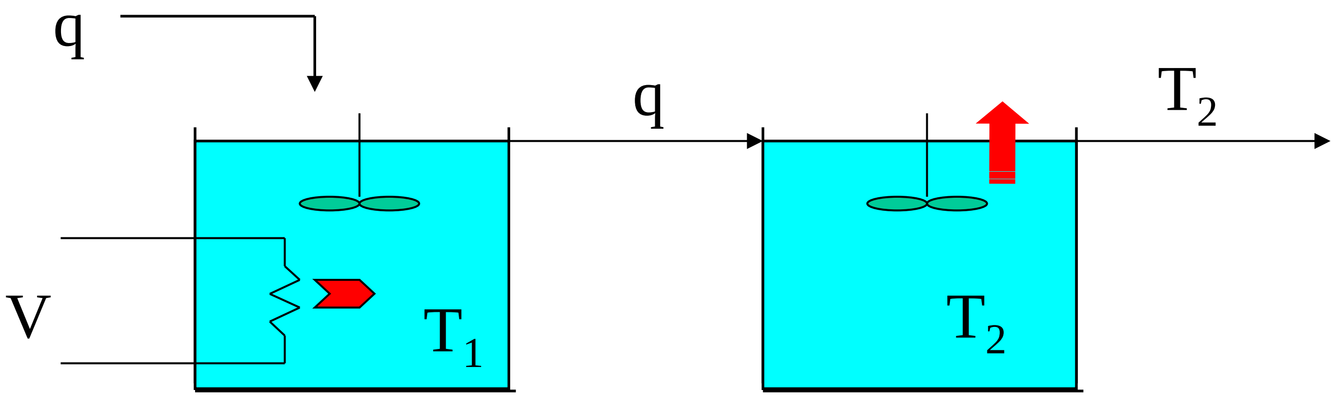


Diagrama de bloques (2)



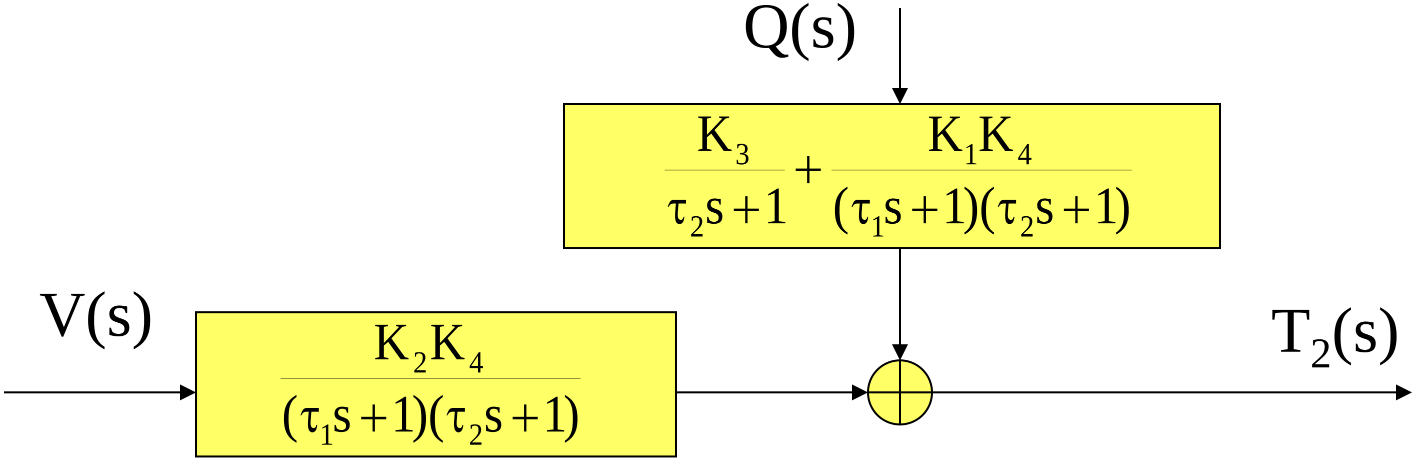
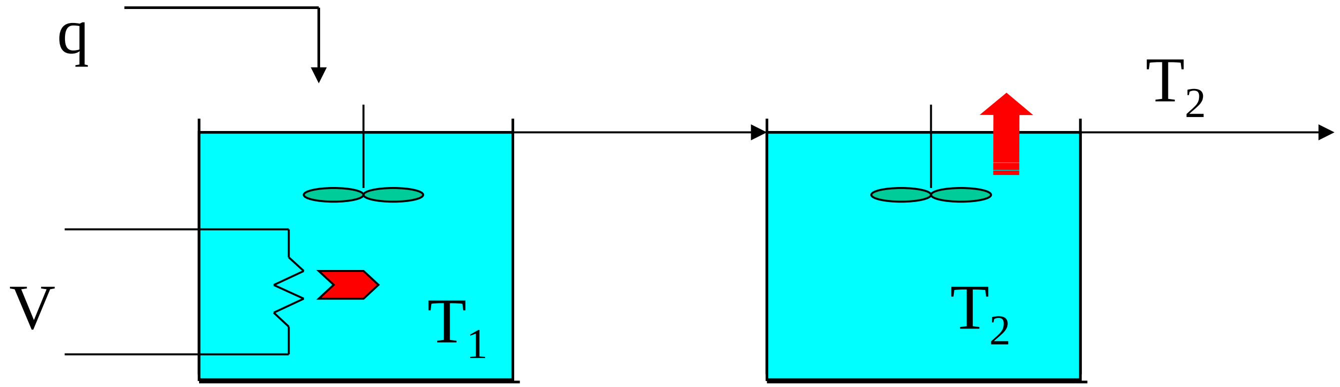
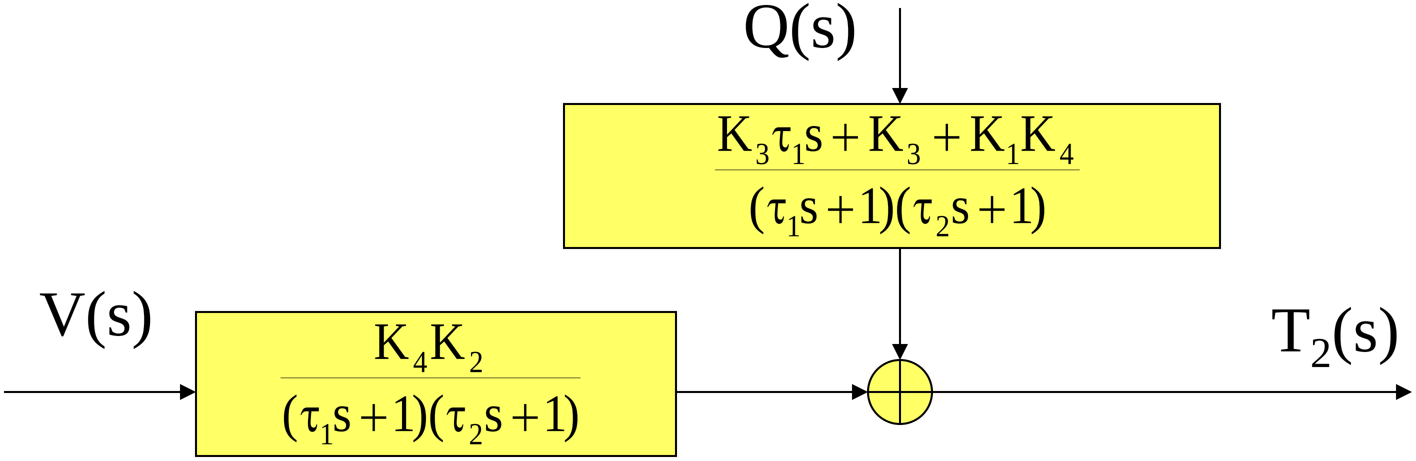


Diagrama de bloques (3)





Alternativa

$$\tau_2 \frac{d\Delta T_2}{dt} + \Delta T_2 = K_4 \Delta T_1 + K_3 \Delta q \Rightarrow \tau_1 \tau_2 \frac{d^2 \Delta T_2}{dt^2} + \tau_1 \frac{d\Delta T_2}{dt} = K_4 \boxed{\tau_1 \frac{d\Delta T_1}{dt}} + \tau_1 K_3 \frac{d\Delta q}{dt}$$

$$\tau_1 \frac{d\Delta T_1}{dt} + \Delta T_1 = K_1 \Delta q + K_2 \Delta V$$

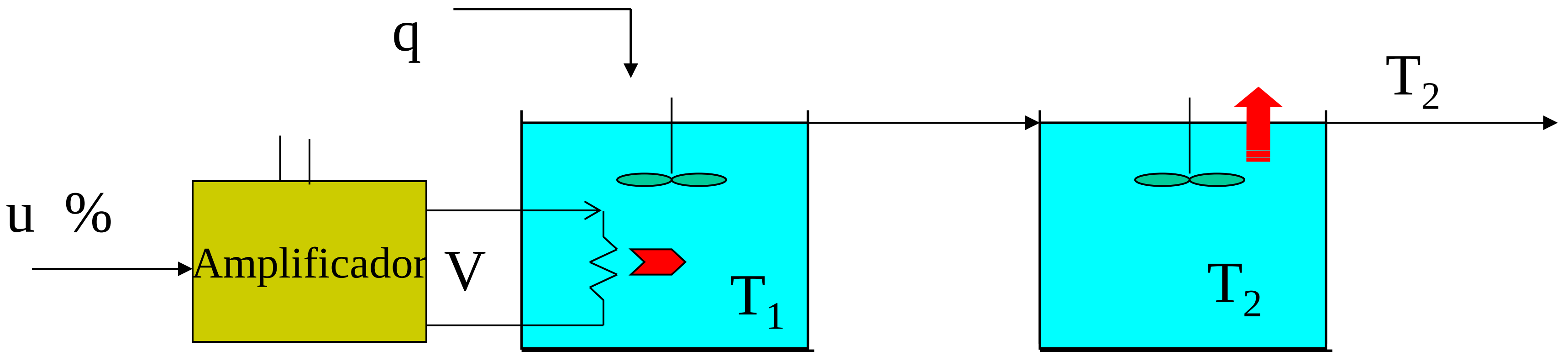
$$\tau_1 \tau_2 \frac{d^2 \Delta T_2}{dt^2} + \tau_1 \frac{d\Delta T_2}{dt} = \boxed{K_4} \left[K_1 \Delta q + K_2 \Delta V - \boxed{\Delta T_1} \right] + \tau_1 K_3 \frac{d\Delta q}{dt}$$

$$K_4 \Delta T_1 = \tau_2 \frac{d\Delta T_2}{dt} + \Delta T_2 - K_3 \Delta q$$

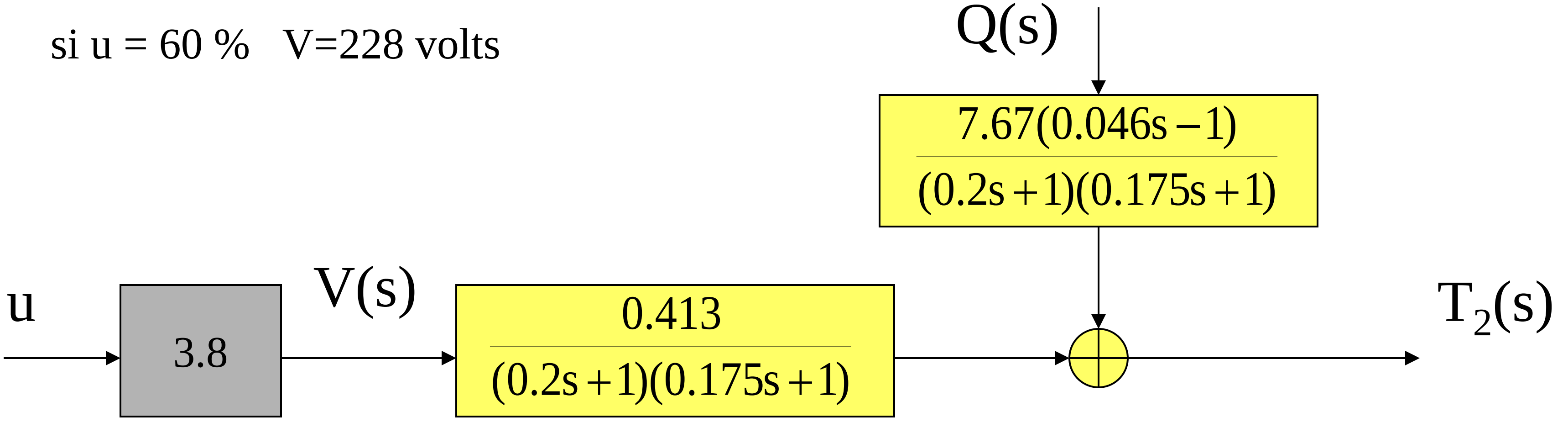
$$\tau_1 \tau_2 \frac{d^2 \Delta T_2}{dt^2} + \tau_1 \frac{d\Delta T_2}{dt} = K_4 K_1 \Delta q + K_4 K_2 \Delta V + \left[-\tau_2 \frac{d\Delta T_2}{dt} - \Delta T_2 + K_3 \Delta q \right] + \tau_1 K_3 \frac{d\Delta q}{dt}$$

$$\tau_1 \tau_2 \frac{d^2 \Delta T_2}{dt^2} + (\tau_1 + \tau_2) \frac{d\Delta T_2}{dt} + \Delta T_2 = \tau_1 K_3 \frac{d\Delta q}{dt} + (K_3 + K_4 K_1) \Delta q + K_4 K_2 \Delta V$$

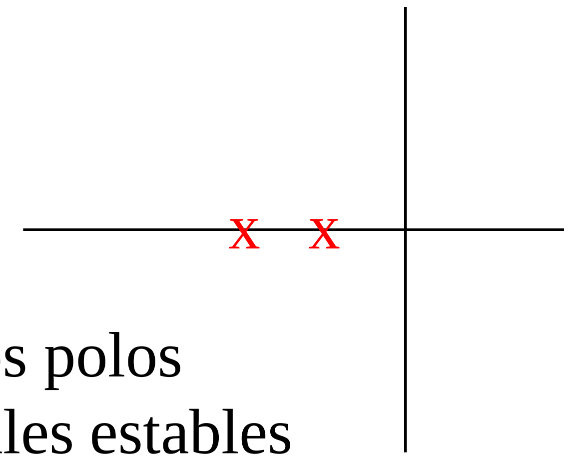
Actuador: Amplificador



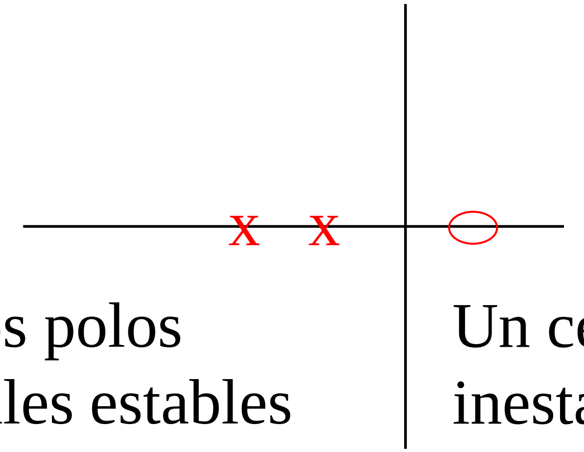
si u = 60 % V=228 volts



Análisis en lazo abierto

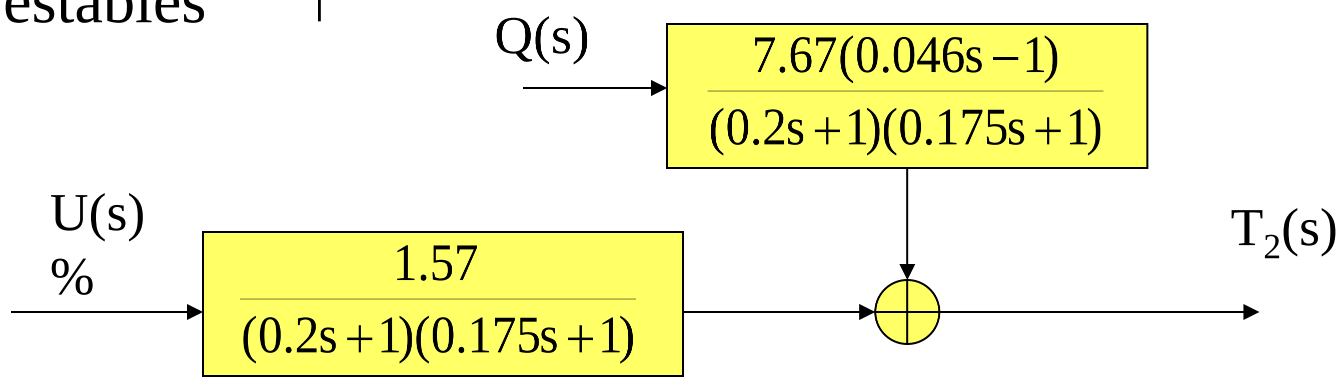


Dos polos reales estables

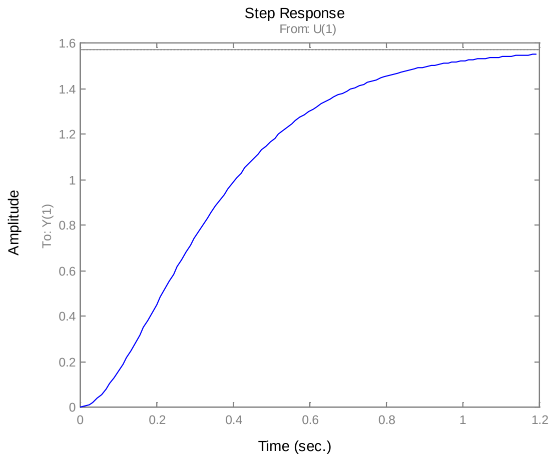


Dos polos reales estables

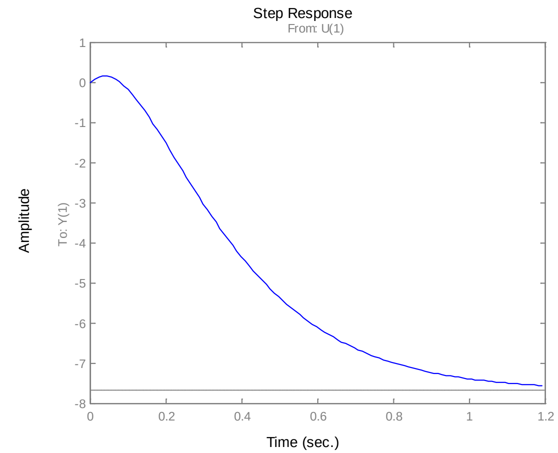
Un cero inestable



Respuestas a un salto en u y q

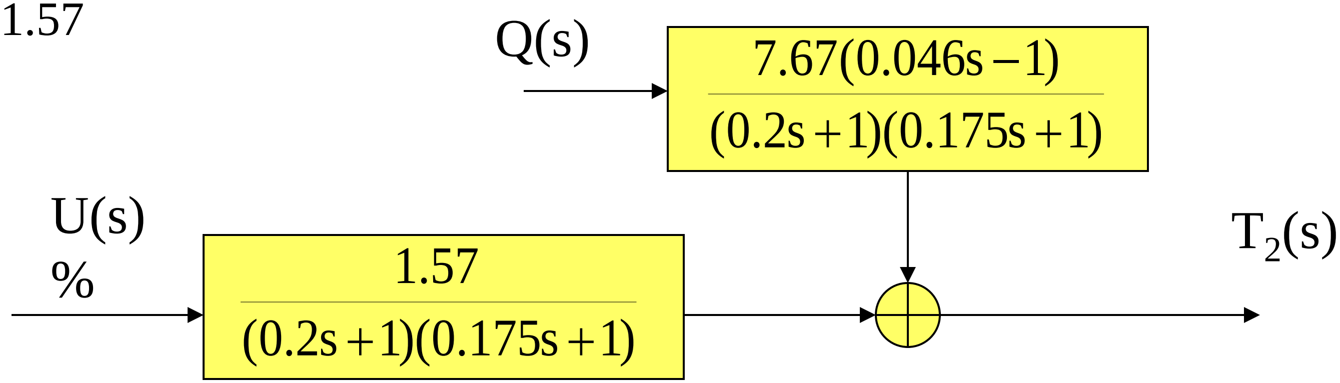


Tiempo
en horas

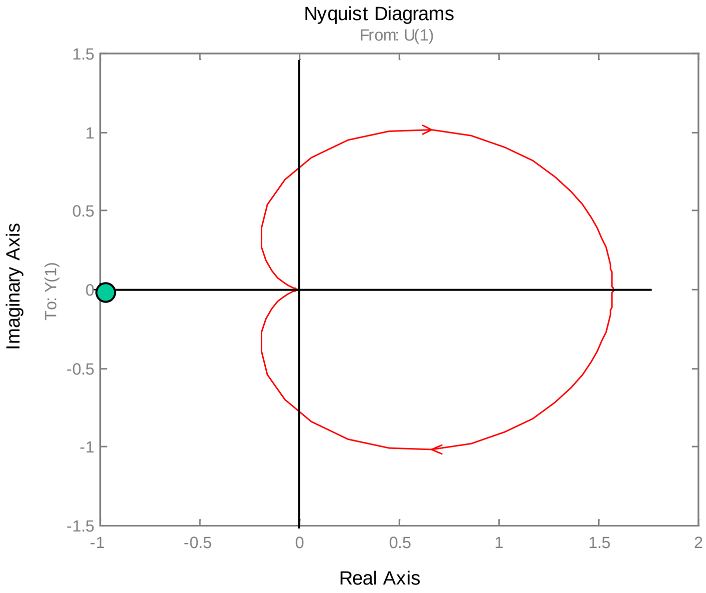


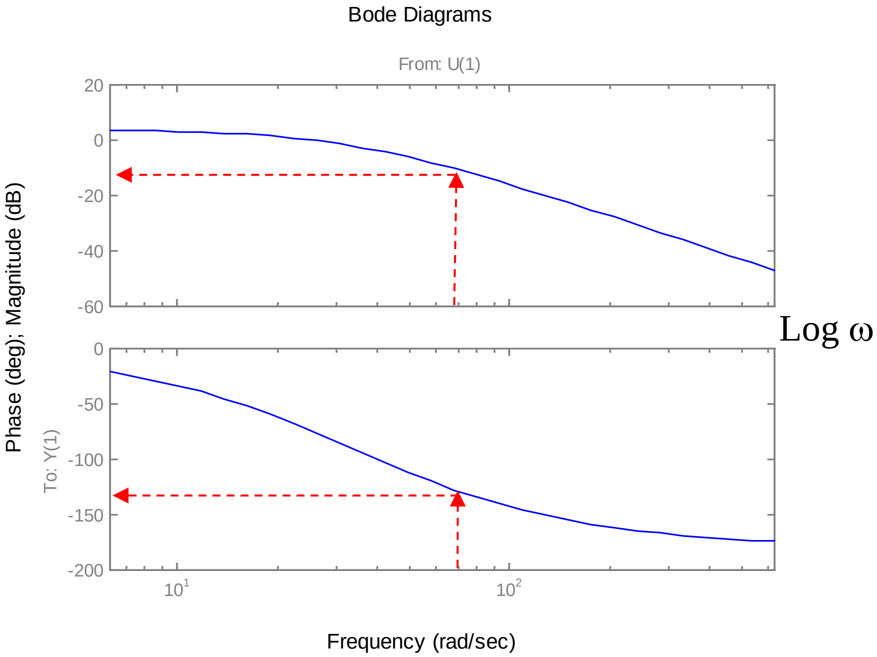
Ganancia -7.67

Ganancia 1.57



Respuesta en frecuencia a u

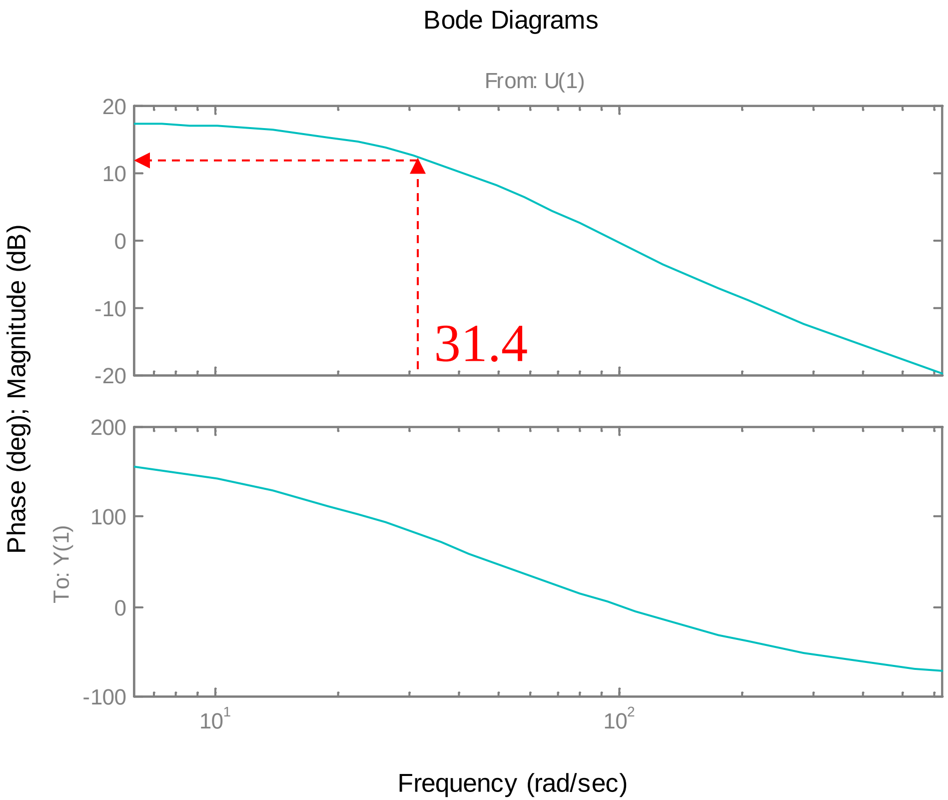




Nyquist

Bode

Respuesta en frecuencia a q



¿Como seria la respuesta a una oscilación sinusoidal en q de 2 m^3/h y periodo 0.2 h?

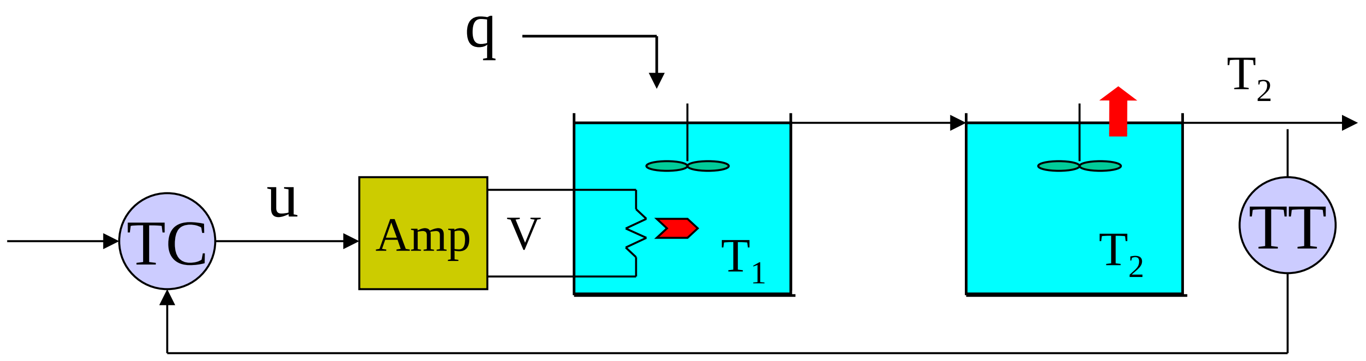
$$\left|D(j\omega)\right| = \frac{7.67\left|0.046\,j\omega - 1\right|}{\left|0.2\,j\omega + 1\right|\left|0.175\,j\omega + 1\right|}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} \approx 31.4 \text{ rad/h}$$

$$\left|\mathrm{D(j31.4)}\right| = 12\ dB = 3.98$$

$$\mathrm{T}_2 = \left|\mathrm{D(j}\omega\mathrm{)}\right| 2 = 7.962\ {}^oC$$

Control en lazo cerrado



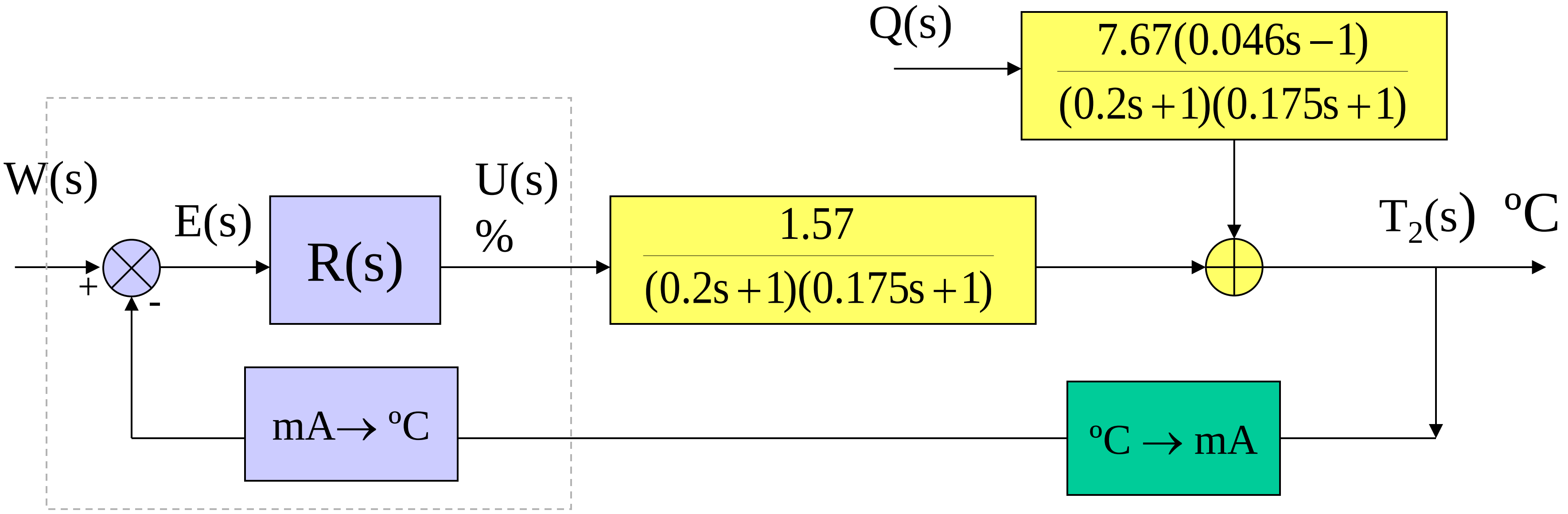
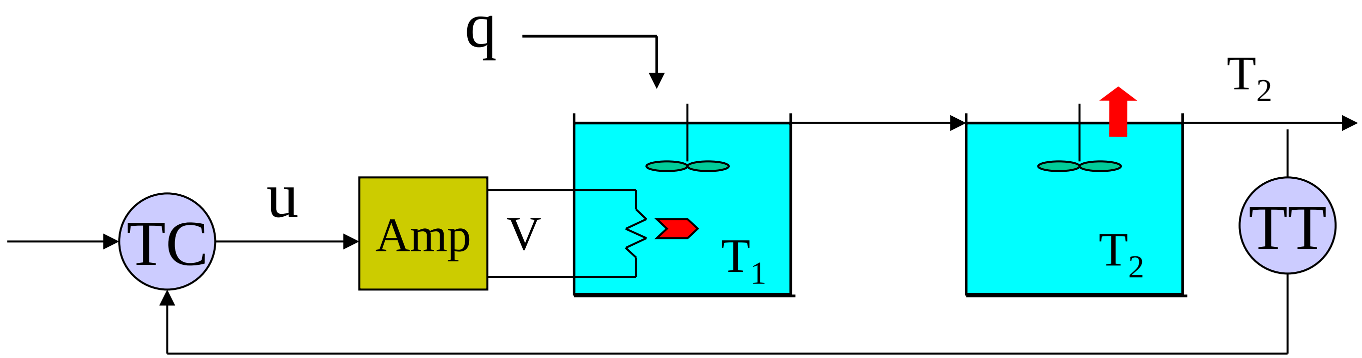
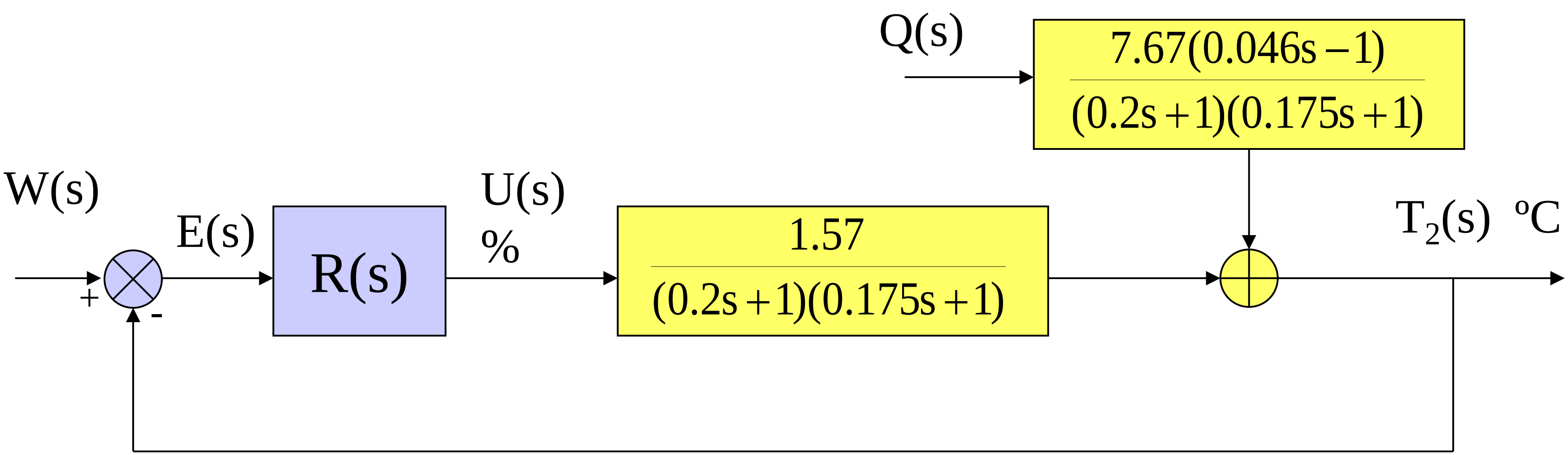
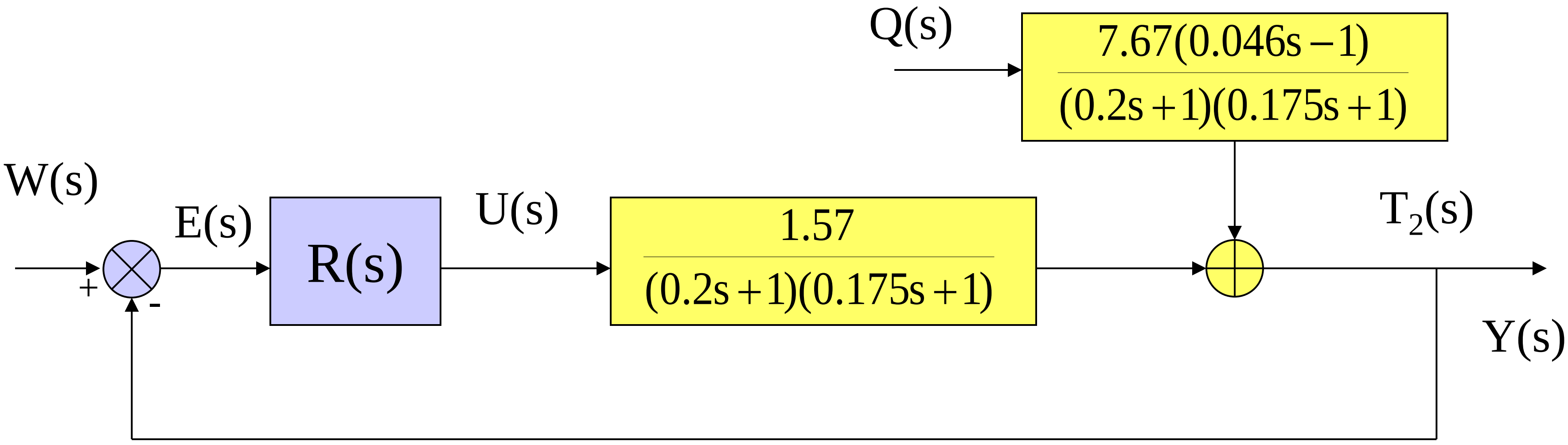


Diagrama en lazo cerrado





Análisis en lazo cerrado

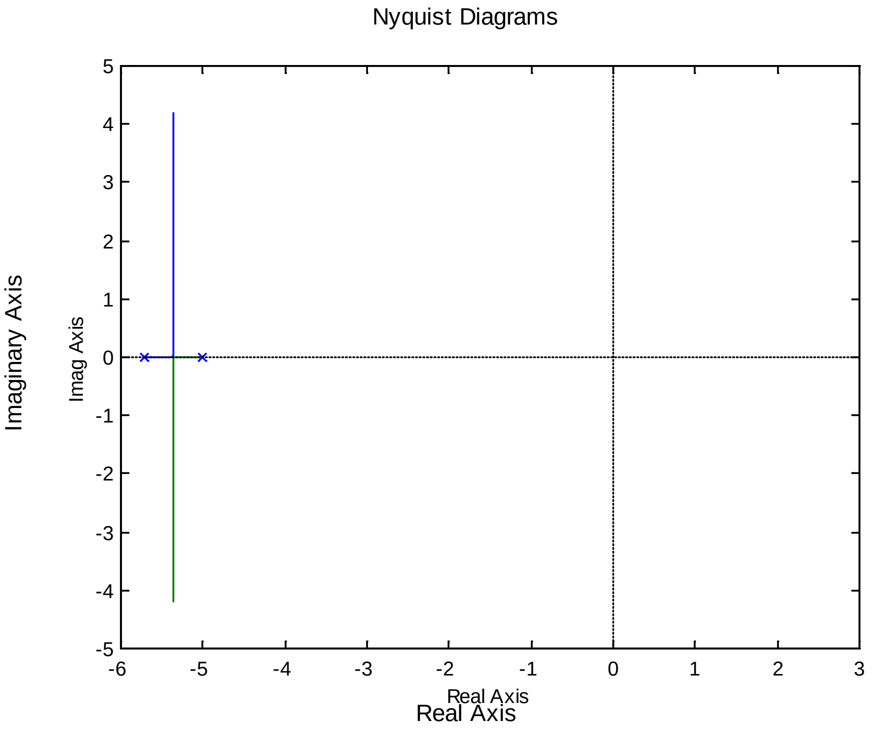


$$T_2(s) = \frac{G(s)R(s)}{1+G(s)R(s)} W(s) + \frac{D(s)}{1+G(s)R(s)} Q(s)$$

Estabilidad
Error estacionario
Respuesta dinámica

Lugar de las raices

¿Para qué valores de la ganancia de un regulador proporcional el sistema tendrá una respuesta sobreamortiguada? ¿Para cuales un sobrepico del 20%?



Ecuación característica

$$1 + G(s)R(s) = 1 + \frac{1.57}{(0.2s+1)(0.175s+1)} K_p = 0$$

$$0.035s^2 + 0.375s + 1 + 1.57K_p = 0$$

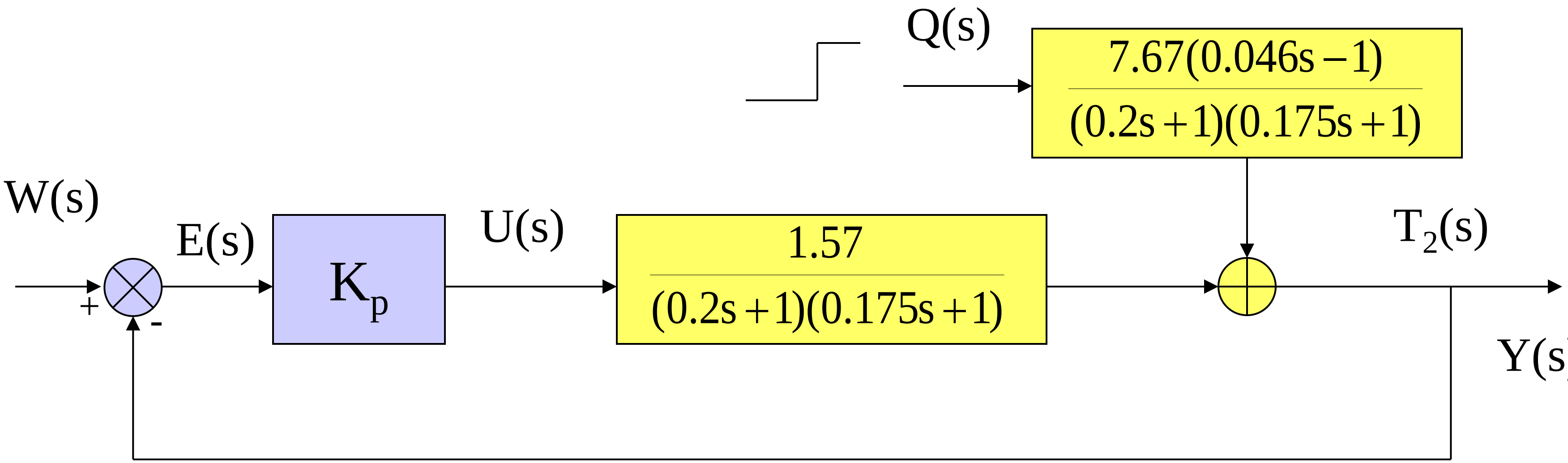
$$s = \frac{-0.375 \pm \sqrt{0.375^2 - 4.\ 0.035(1+1.57K_p)}}{2\ \ 0.035}$$

$$0.375^2 - 4.\ 0.035(1+1.57K_p) = 0 \qquad K_p = 0.00284$$

$$M_p = 100e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \text{ en } \% \qquad 1.6094 = \frac{\pi\delta}{\sqrt{1-\delta^2}} \quad \Rightarrow \quad \delta = 0.456$$

$$s^2 + 2\delta\omega_n s + \omega_n^2 = 0, \quad 2\ \ 0.456\ \sqrt{\frac{1+1.57K_p}{0.035}} = \frac{0.375}{0.035} \Rightarrow K_p = 2.44$$

Error estacionario



Si q experimenta un cambio en salto de 2 m^3/h ¿Como será el error estacionario con un regulador P de ganancia 2?

Error estacionario

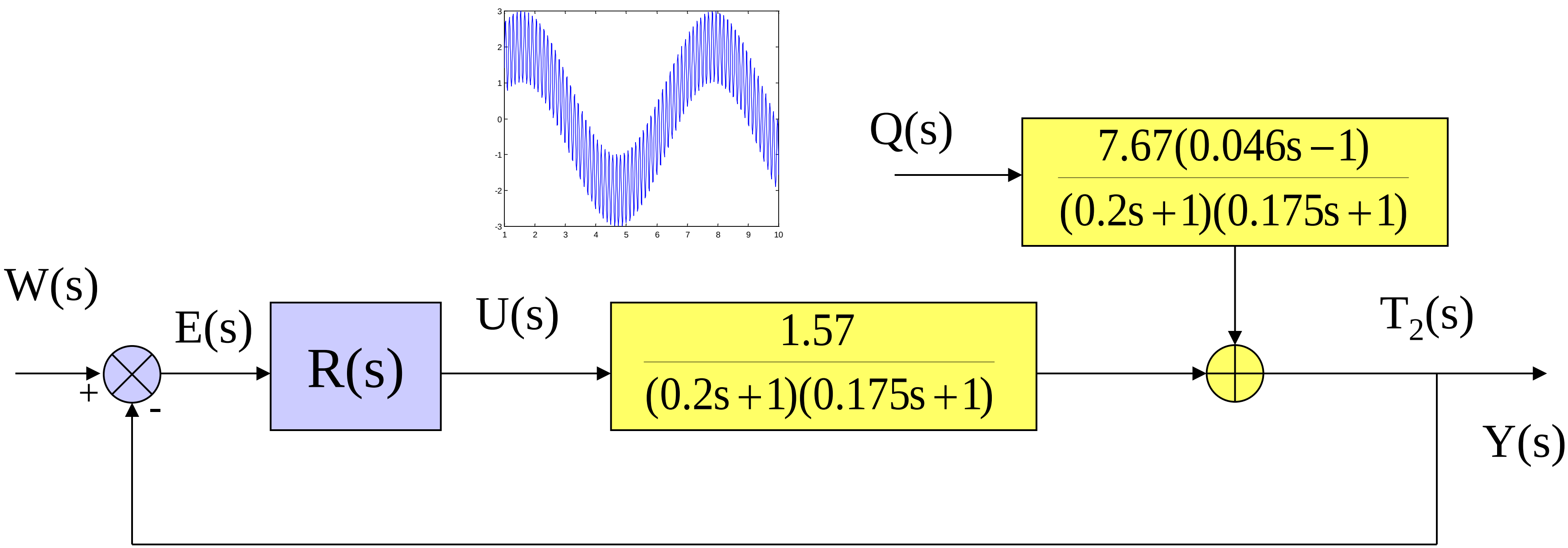
$$E(s) = \frac{1}{1+G(s)R(s)}W(s) - \frac{D(s)}{1+G(s)R(s)}Q(s)$$

$$E(s) = -\frac{\dfrac{7.67(0.046s-1)}{(0.2s+1)(0.175s+1)}}{1+\dfrac{1.57}{(0.2s+1)(0.175s+1)}K_p}Q(s) =$$

$$E(s) = -\frac{7.67(0.046s-1)}{(0.2s+1)(0.175s+1)+1.57K_p}\frac{2}{s}$$

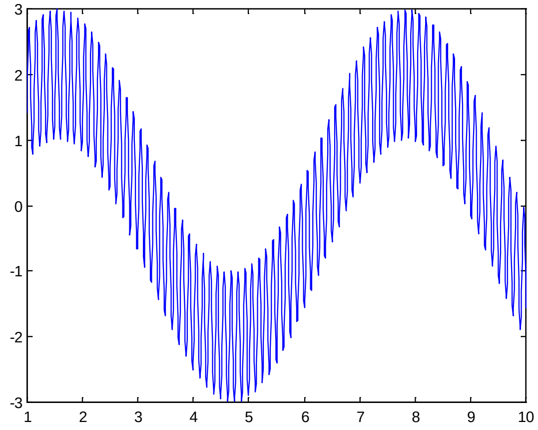
$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{2 \quad 7.67}{1+1.57K_p} = 3.7^{o}\,C$$

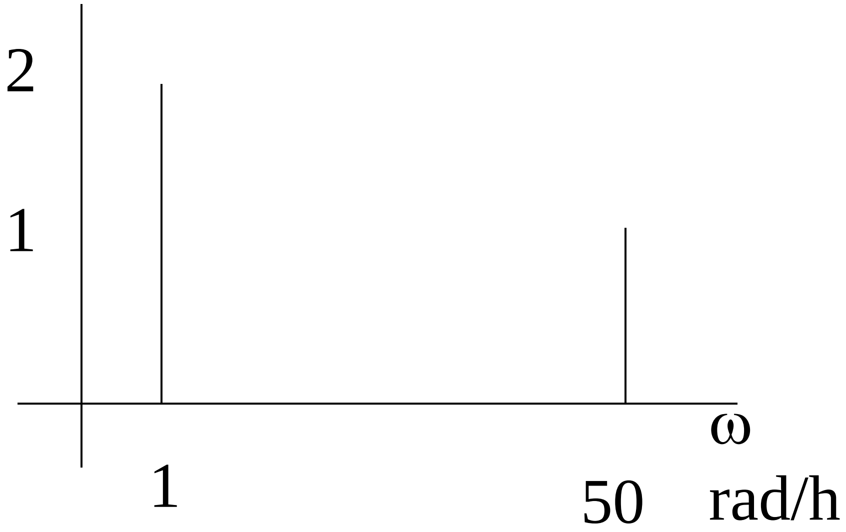
Respuesta a perturbaciones



Si con un regulador proporcional de ganancia 1, el caudal varia como en la figura, ¿Como será la respuesta del sistema en temperatura?

Respuesta a perturbaciones





$$T_2(s) = \frac{G(s)R(s)}{1+G(s)R(s)} W(s) + \frac{D(s)}{1+G(s)R(s)} Q(s)$$

$$T_2(j\omega) = \frac{D(j\omega)}{1+G(j\omega)K_p} Q(j\omega)$$

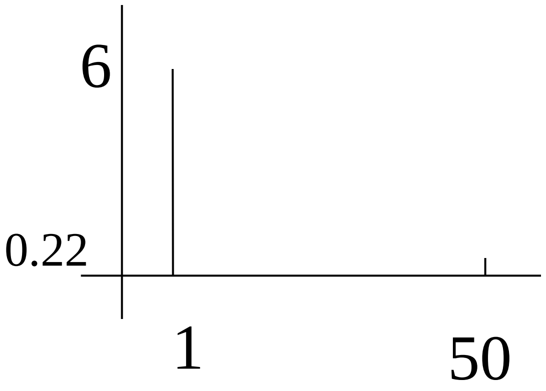
Respuesta a perturbaciones

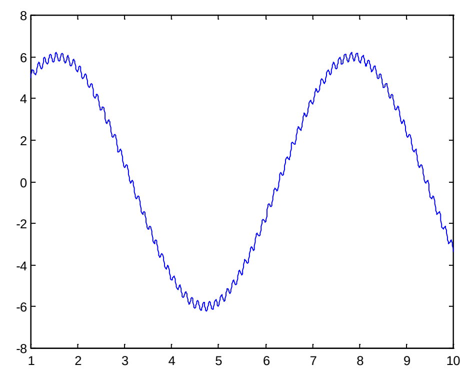
$$T_2(j\omega) = \frac{D(j\omega)}{1+G(j\omega)K_p} Q(j\omega)$$

$$\left| \frac{7.67(0.046\,j\omega - 1)}{(0.2\,j\omega+1)(0.175\,j\omega+1)+1.57K_p} \right|$$

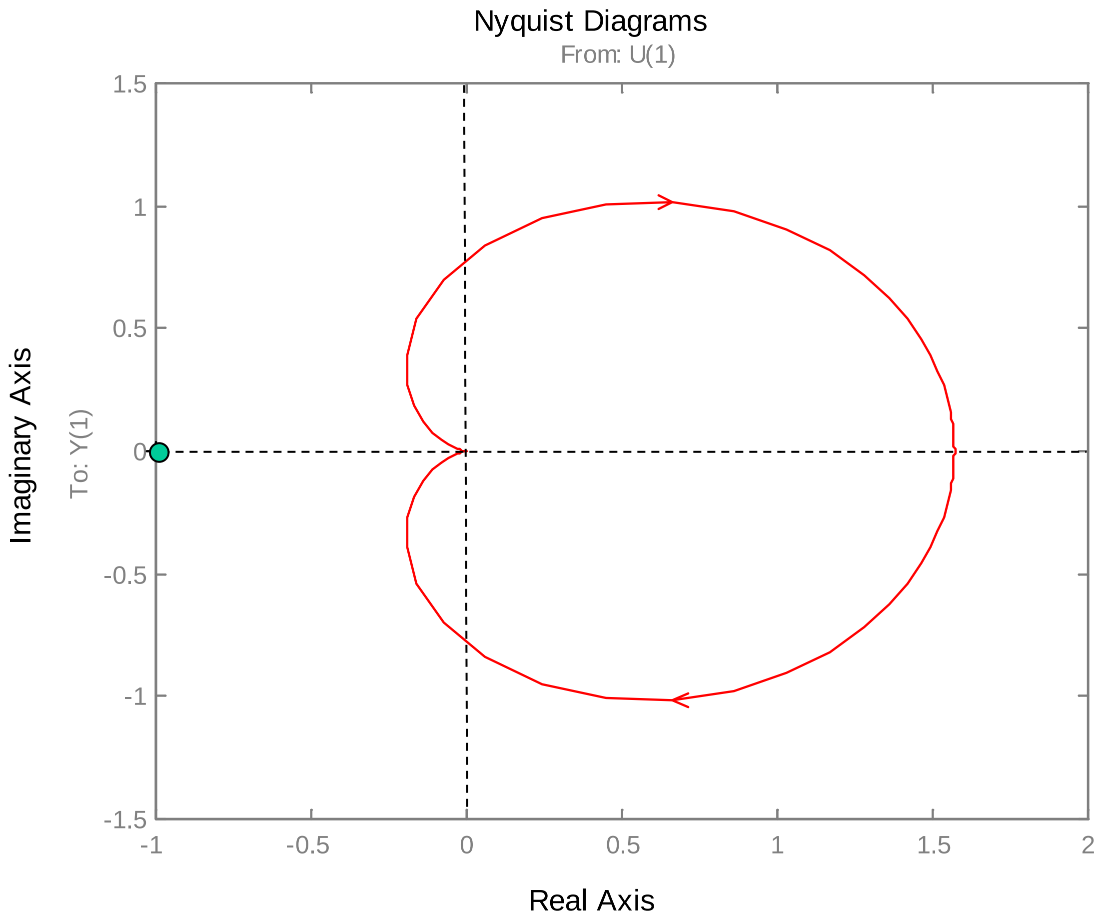
$$\omega = 1 \qquad \frac{7.67\sqrt{(0.046^2+1)}}{\sqrt{(2.57-0.035)^2+0.375^2}} \approx 2.98$$

$$\omega = 50 \qquad \frac{7.67\sqrt{(0.046^2 50^2+1)}}{\sqrt{(2.57-0.035\,50^2)^2+0.375^2 50^2}} \approx 0.22$$





Nyquist con P

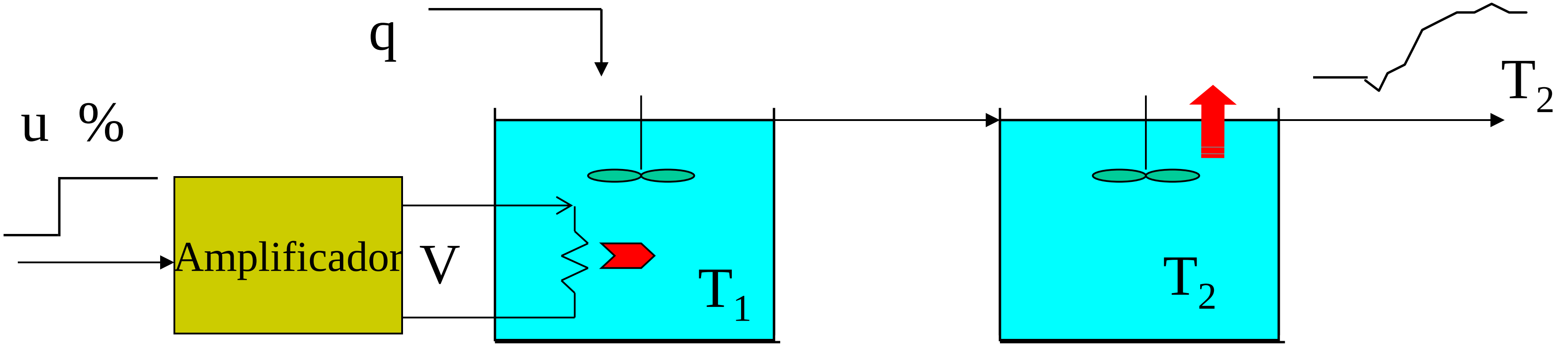


Estudiar la robustez del lazo con P

¿Como sintonizar un regulador?

- Criterios:
  - No tener error estacionario frente a cambios escalon de la referencia o q
  - Atenuación 1/4 frente a cambios de q
  - Minimizar el error acumulado frente a cambios en la referencia
  - Obtener un comportamiento como un sistema de primer orden con tiempo de asentamiento 0.5 min
  - etc.

Identificación



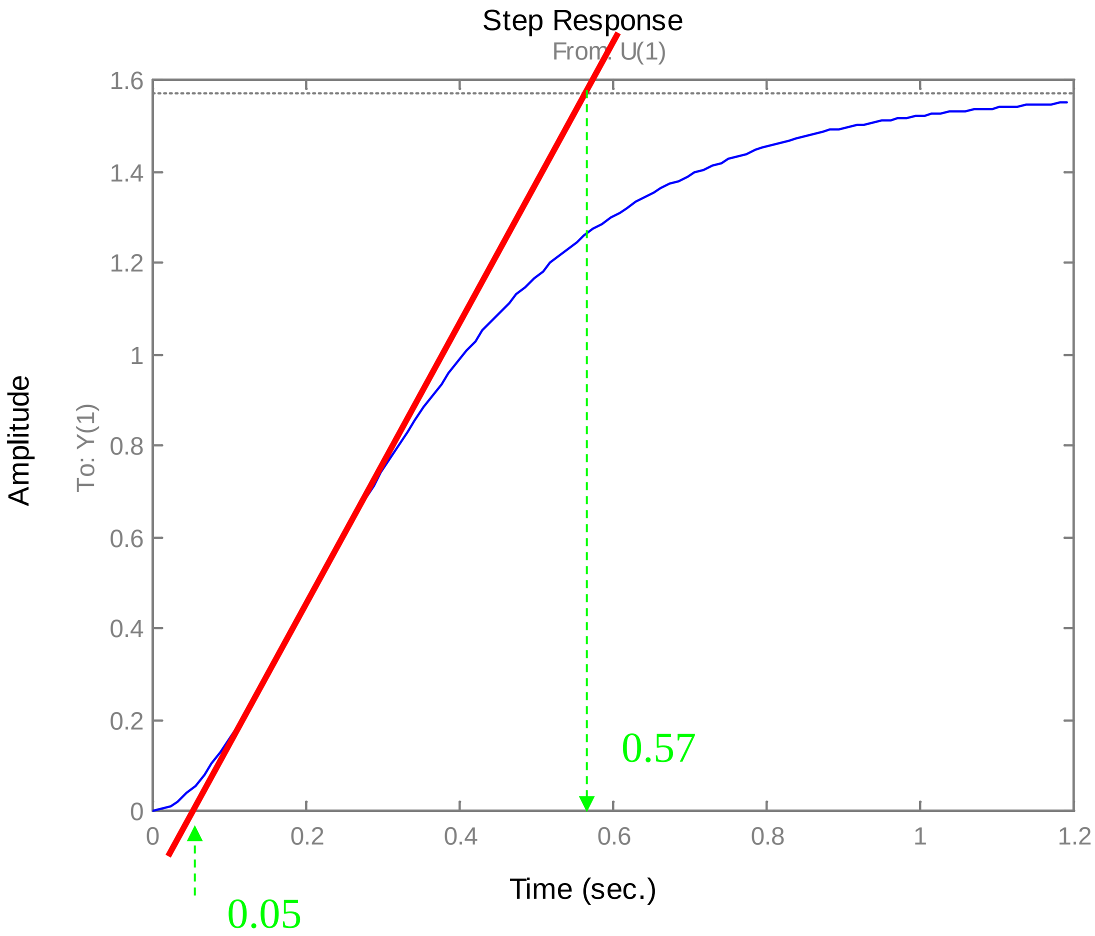
Dos experimentos:

•Cambio en u con q cte.

Ajuste con funciones de primer orden mas retardo

$$T_2(s) = \frac{Ke^{-ds}}{\tau s + 1} U(s)$$

Salto unitario en u



Qué modelo aproximado puede obtenerse?

$$K = \frac{1.57}{1} = 1.57$$

$$d = 0.05$$

$$\tau = 0.57 - 0.05 = 0.52$$

$$\frac{1.57e^{-0.05s}}{0.52s + 1}$$

Sintonía

Criterio: Regulador mas sencillo que no tenga error estacionario frente a cambios salto en la referencia y que rechace perturbaciones atenuandolas a 1/4 del valor anterior

Tabla de Ziegler Nichols

Tipo	**Ganancia K_p**	**Tiempo integral**	**Tiempo derivativo**
P	τ / (K d)		
PI	0.9τ /(K d)	3.33 d	
PID serie	1.2τ /(K d)	2 d	0.5 d

K expresado en % / %

Sintonía

Si la medida de temperatura se ha hecho con un transmisor calibrado en el rango 0 - 50 ºC:

$$K = \frac{1.57 \; 100/50 \quad \%}{1 \quad \%} = 3.14 \; \%/\%$$

$$\frac{3.14e^{-0.05s}}{0.52s + 1}$$

$$K_p = 0.9\tau / Kd = 0.9 \; 0.52/3.14 \; 0.05 = 2.98$$

$$T_i = 3.33d = 0.16 \text{ h} = 10 \,\text{min}$$