Introduction to scheduling of batch processes

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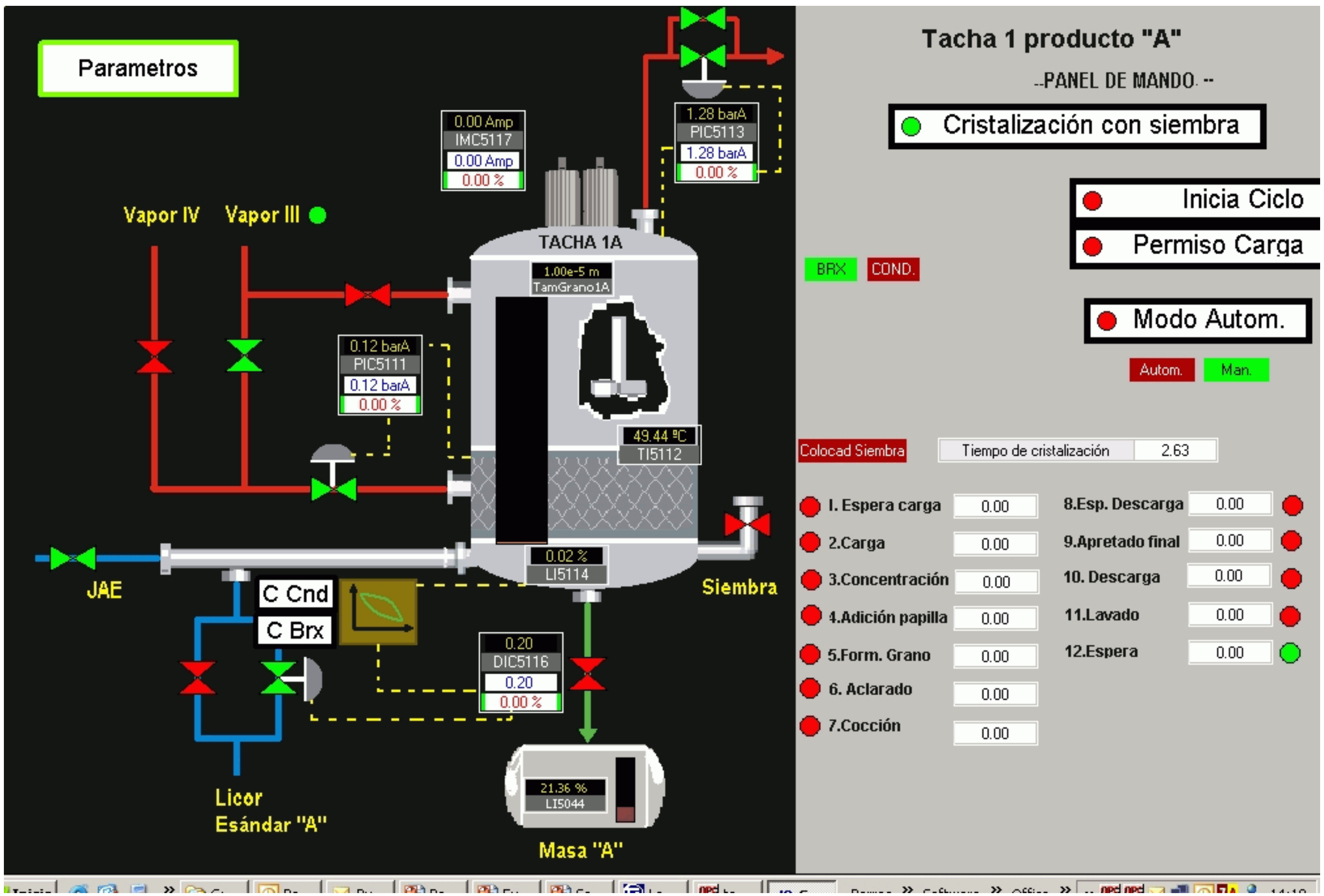
Outline

- Batch processes and batch plants
- Basic concepts of scheduling
- How to formulate scheduling problems
- Solution with optimization tools

Batch plants

- As the demand of high added value products of limited production (fine-chemicals, pharmaceuticals, food, certain polymers,....) is growing, the interest for batch plants in the process industry has increased
- In the same direction, the concept of multiproduct flexible production, where the equipment is re-used in order to manufacture different products according to a demand-driven scheme, has risen this interest.
- The use of batch processes requires a careful production planning and scheduling, that determines which products must be manufactured or processed, in which process units, in what order as well as the starting and ending times in each process unit.

Batch units



Operation

Load

Sequence of internal operation and stages in the unit

Unload

Recipe for the operation

Basically, control problems

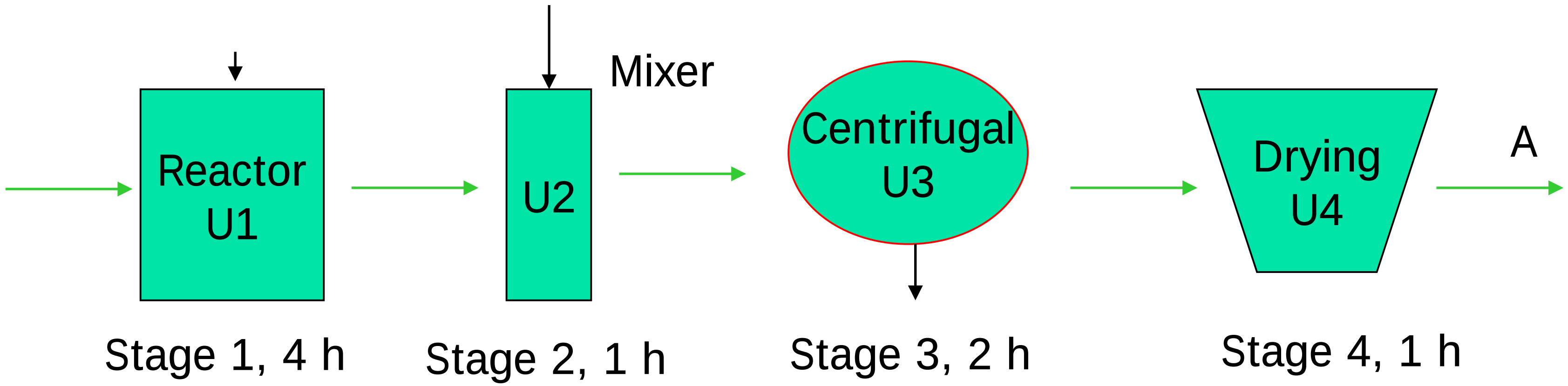
Operation of batch plants

When a firm operates with batch units, a key problem is to determine when each one should be started and unloaded and which products must process, so that a certain amount of products is manufactured satisfying constraints on energy, quality, storage space, etc, and optimizing some adequate criterion.

Single product manufacturing

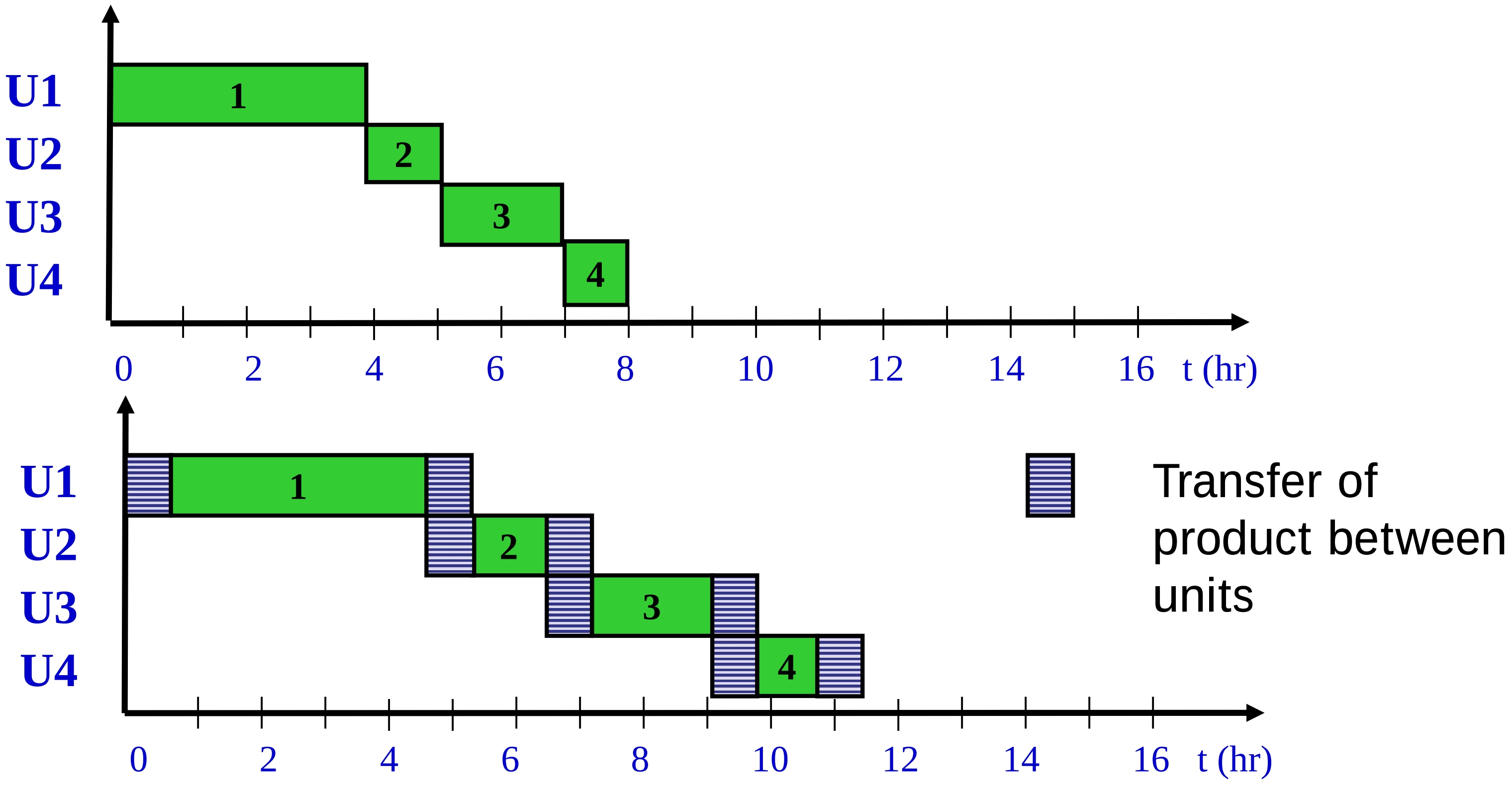
Usually the manufacturing of a product implies several stages that take place in different batch process units according to a certain recipe.

Example:

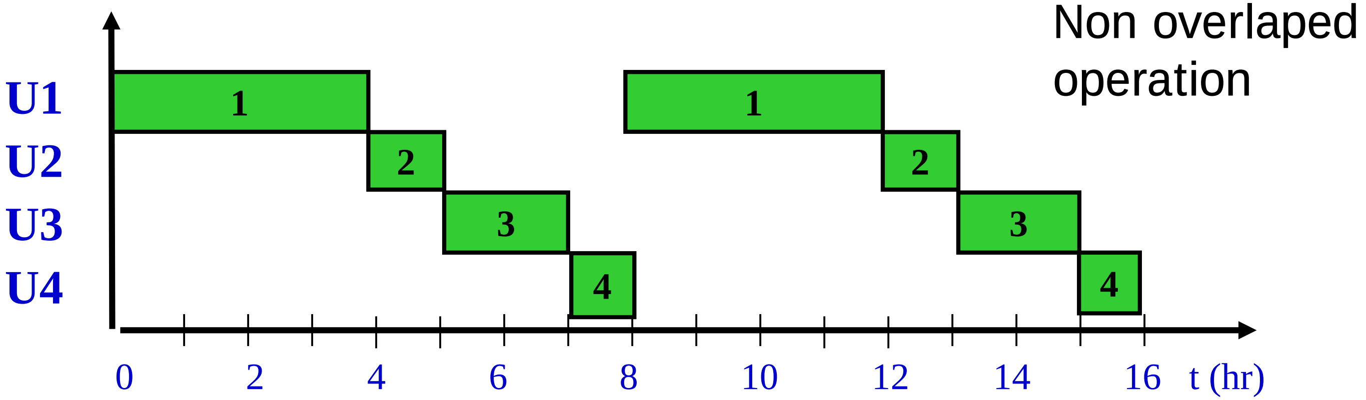


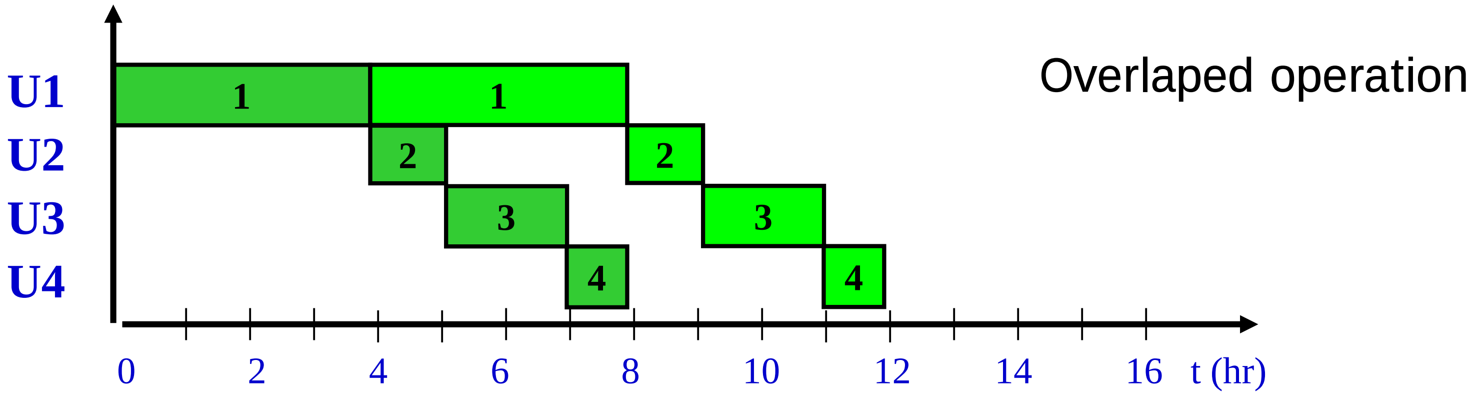
In the example, manufacturing of product A cover four successive stages (each one in a different batch unit, lasting the indicated times)

Gantt diagram

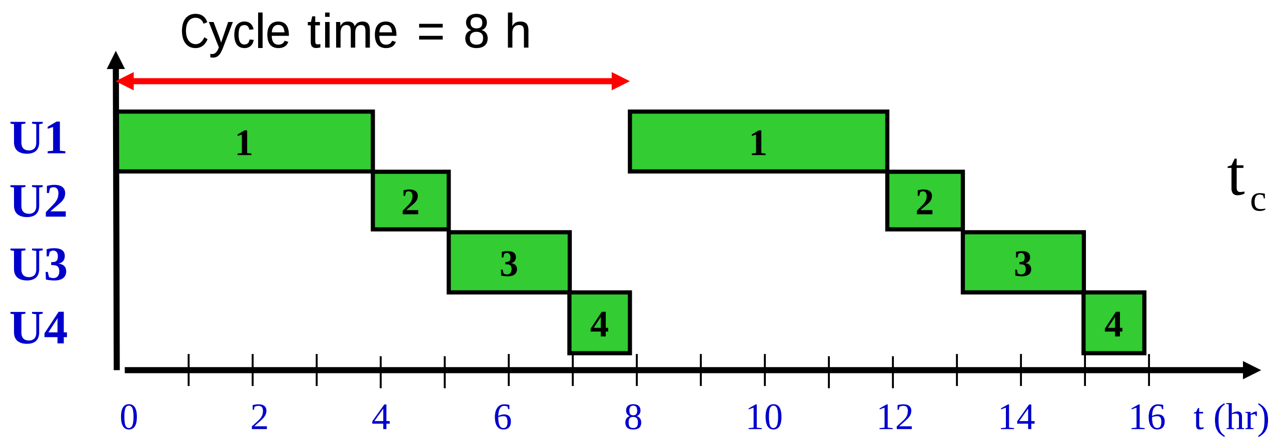


Gantt diagram



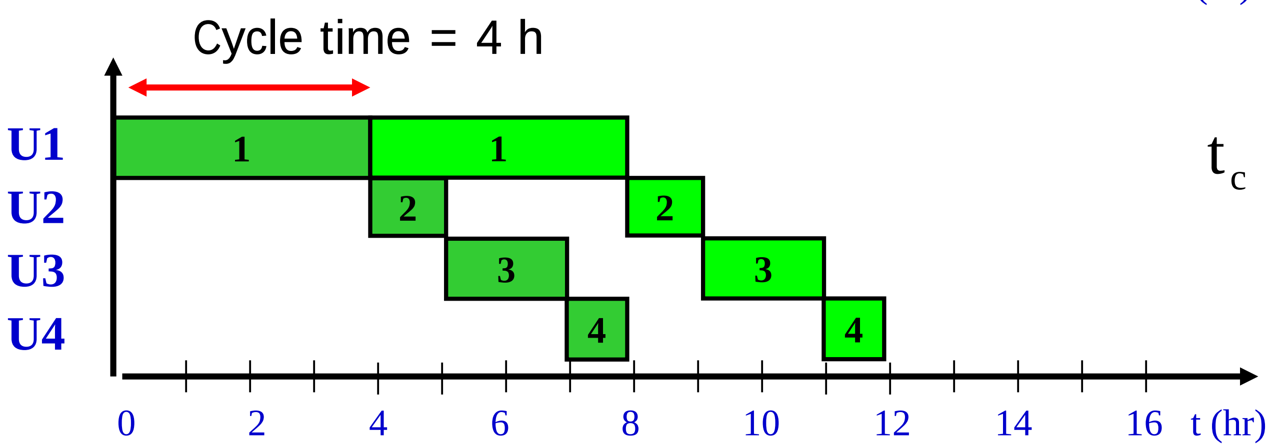


Cycle Time



$$t_c = \sum_{j=1}^{M} \tau_j$$

M = number of stages



$$t_c = \max_{j=1,M} \{\tau_j\}$$

Time interval between the start of two consecutive cycles

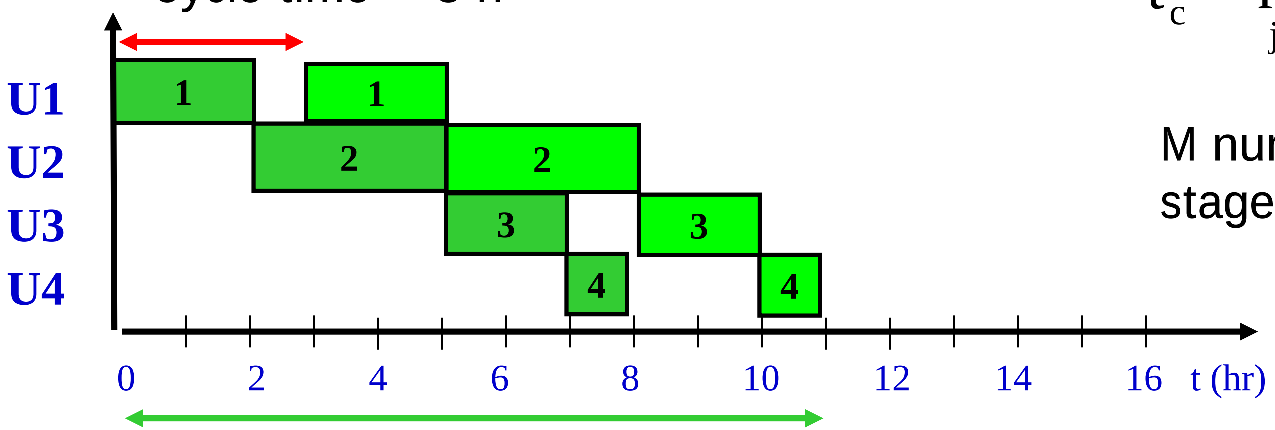
Cycle Time

Another example. Notice the duration of stages 1 and 2

Processing times in the batch units:

U1= 2h , U2 = 3h, U3 = 2h, U4 = 1h   No product storage

Cycle time = 3 h



Makespan = 11 h

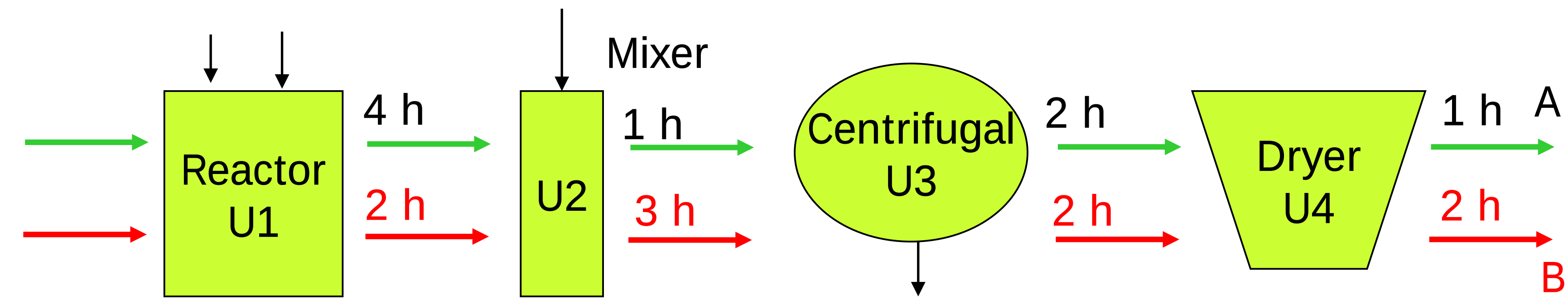
$$t_c = \max_{j=1,M} \{\tau_j\}$$

M number of stages

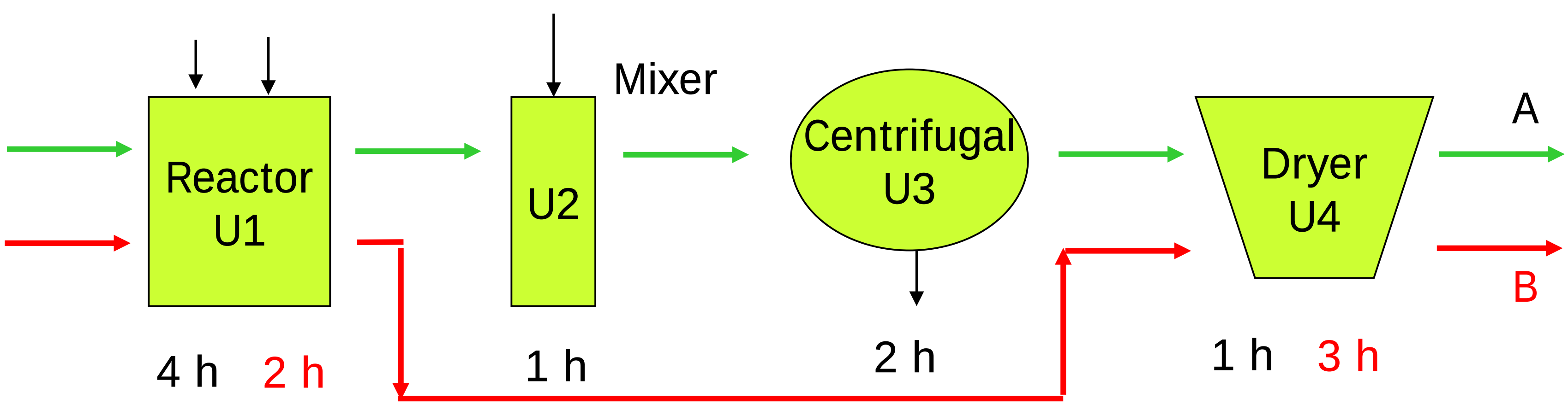
Makespan: Total time required to produce a certain number of lots (example 2)

Multiple products manufacturing

Flowshop plant: Each product follows all stages in the same order (**multiproduct** plant)



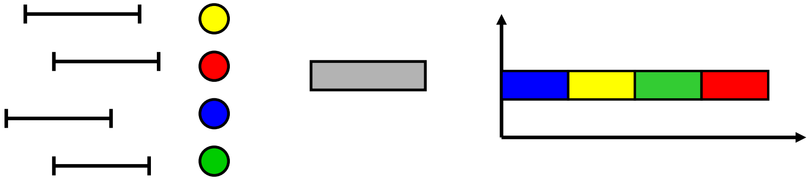
Jobshop plant: Not all products use all stages or follow the same sequence (**multipurpose** plants)



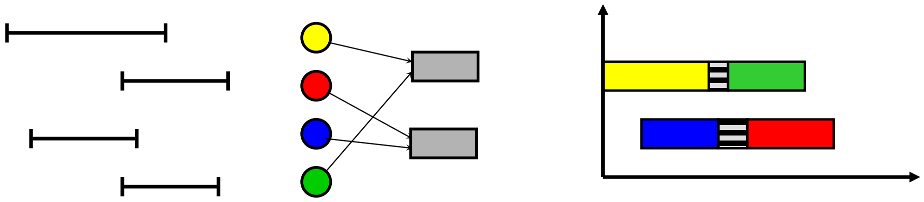
Multiproduct manufacturing

Each stage can have one or several batch units in parallel

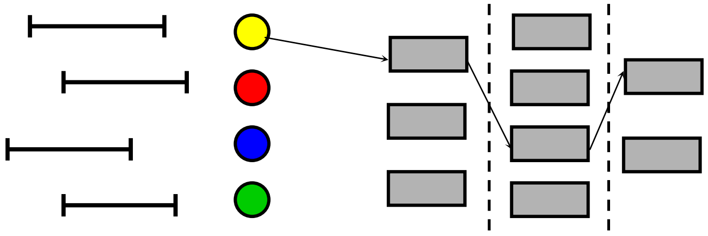
Single unit or machine



Several units or machines in a Single-stage manufacture

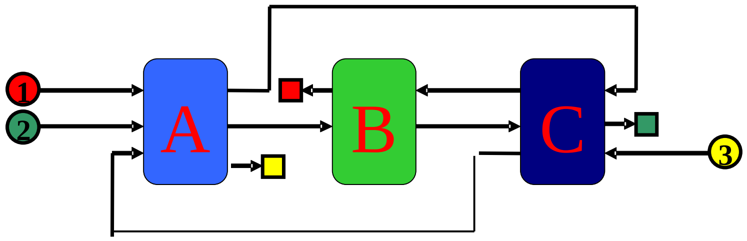


Flow-shop with Parallel units and Multi-stage manufacturing



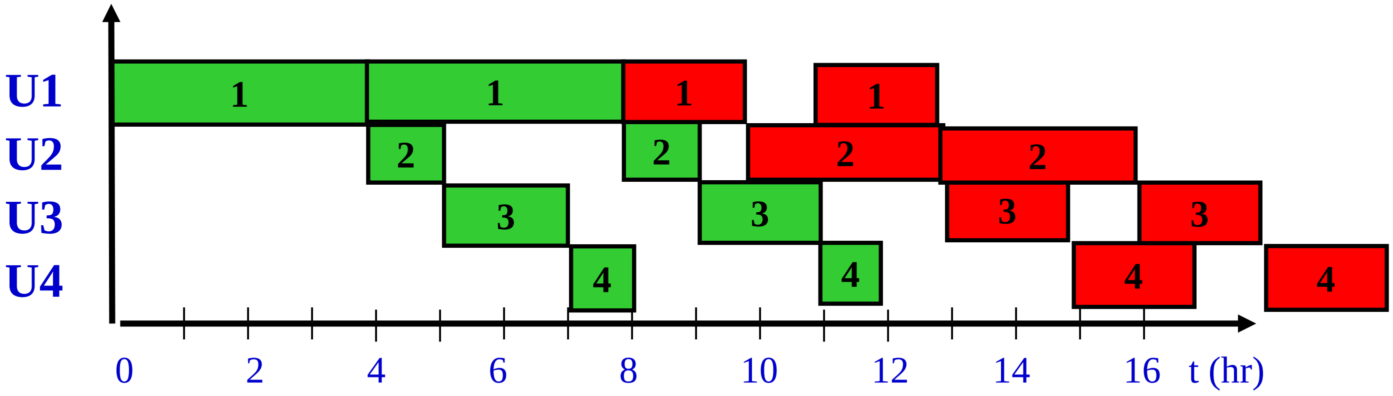
All products use all stages following the same sequence

Job-shop



Not all products use all stages or follow the same sequence

Example, two products



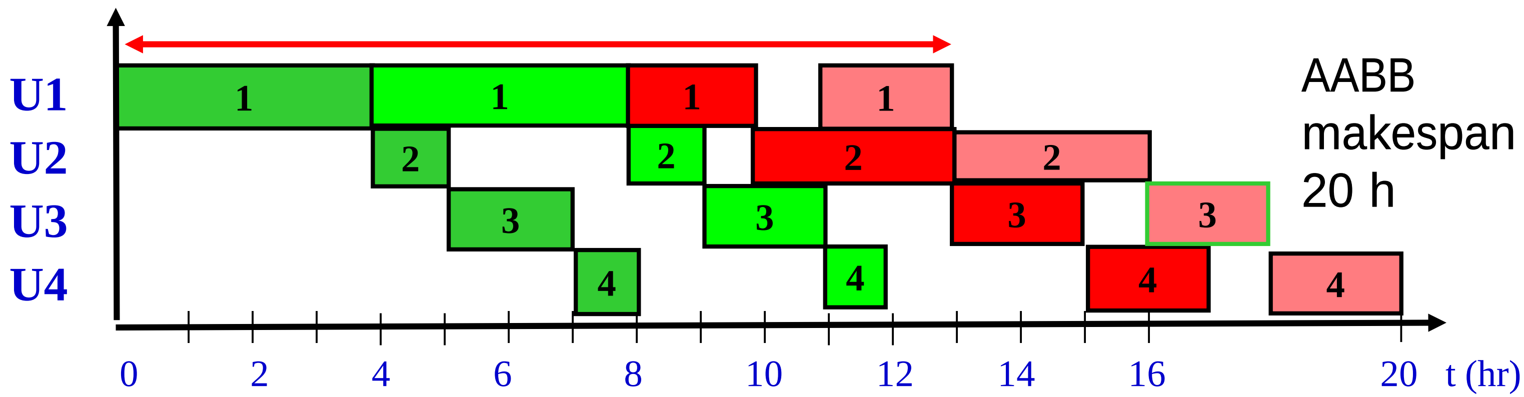
Campaign: manufacturing of a certain number of lots of the different products

Example: Campaign AABB

Types of Campaigns

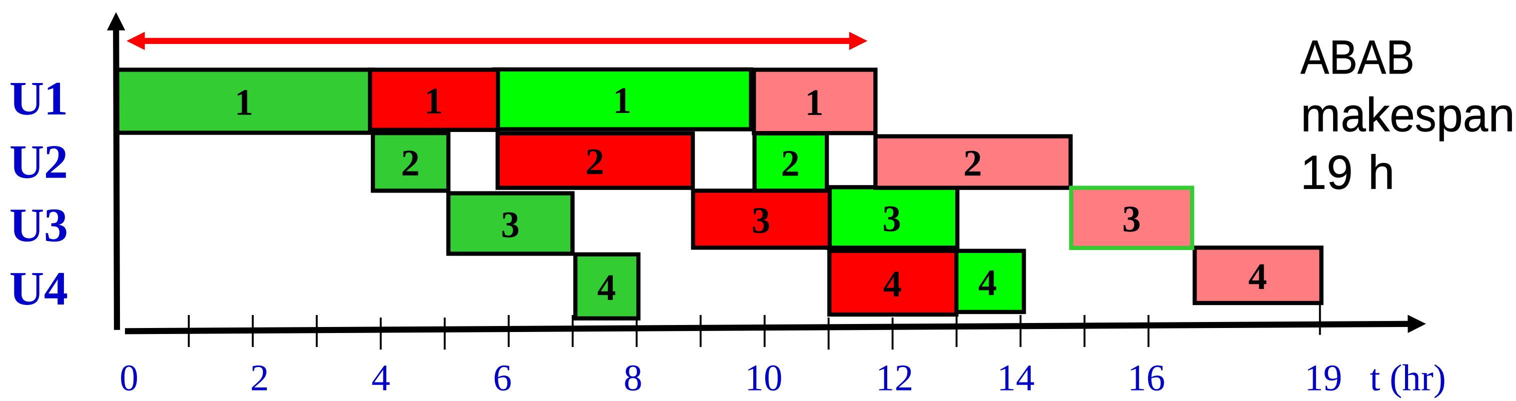
Campaign cycle time 13h

single product campaign **SPC**



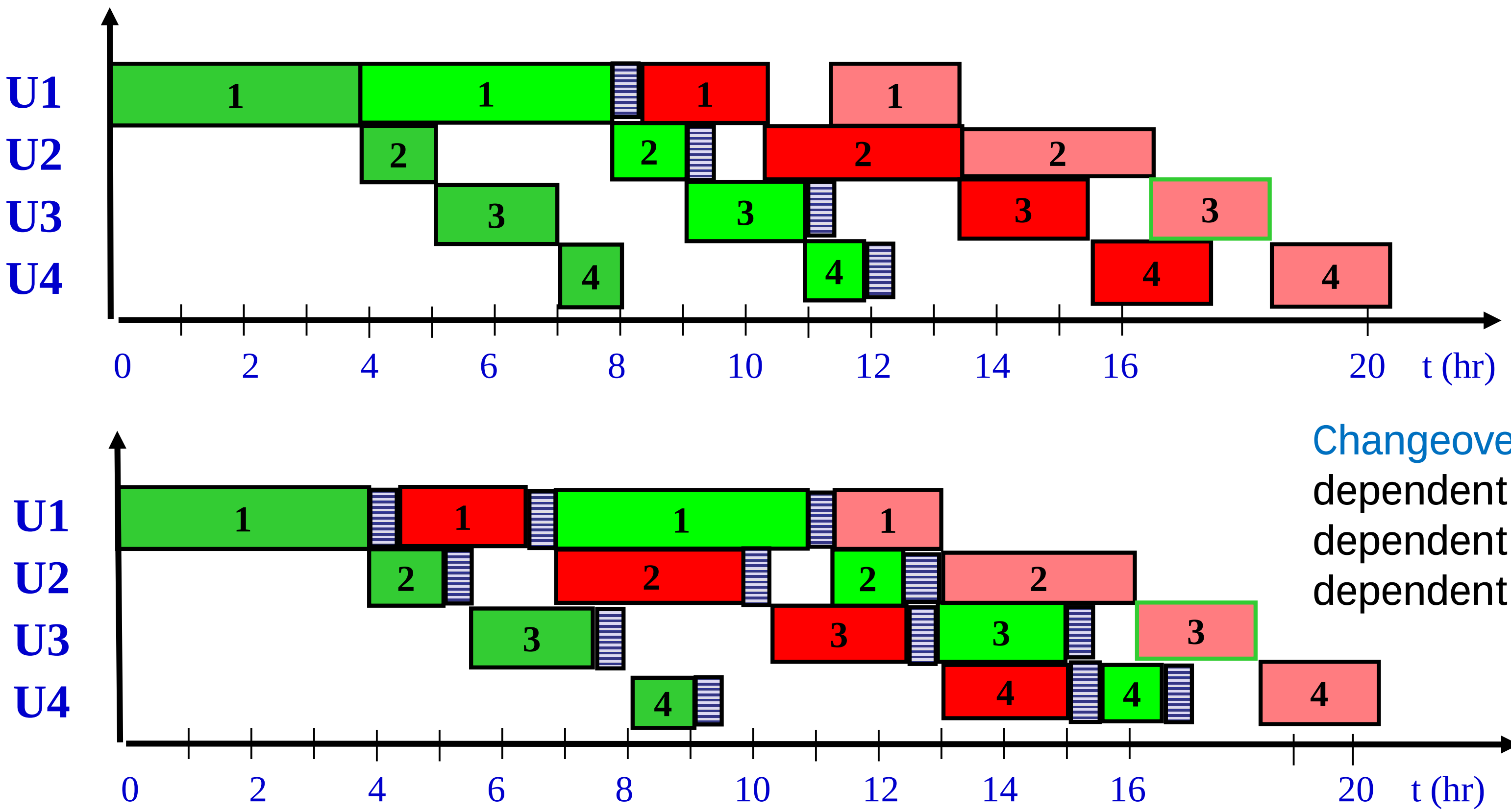
Campaign cycle time 12h

mixed product campaign **MPC**



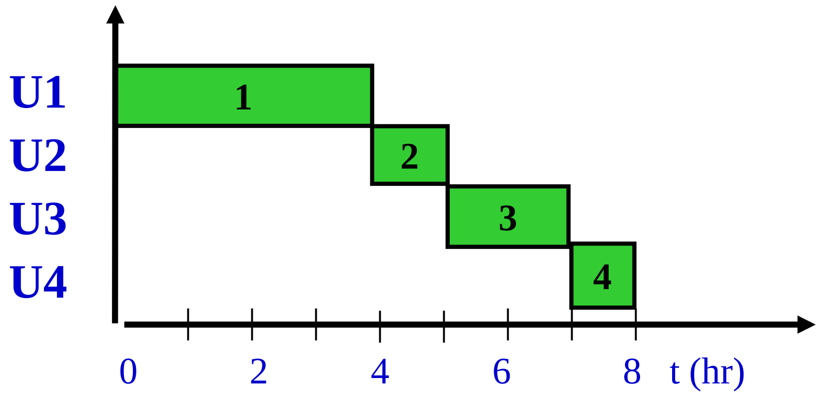
Types of Campaigns

Generally speaking mix product campaigns are more efficient that single product ones, but this will depend on the cleaning times associated to the switching of products (changeovers)



Changeovers: fix, unit dependent, frequency dependent, product dependent...

Storage types



•Storage in the same unit: There are no intermediate storage tanks but the product can be kept in the same unit in which it was processed (NIS non-intermediate storage

•No waiting time: There is no intermediate product storage and, once finished, the product cannot be maintained in the unit (ZW zero wait)

•Unlimited Intermediate Storage: There exist intermediate storage tanks of unlimited capacity (UIS unlimited intermediate storage) or finite capacity (FIS Finite Intermediate Storage) (shared or not)

$$t_c = \max_{j=1,M} \sum_{i=1}^{N} n_i \tau_{ij}$$

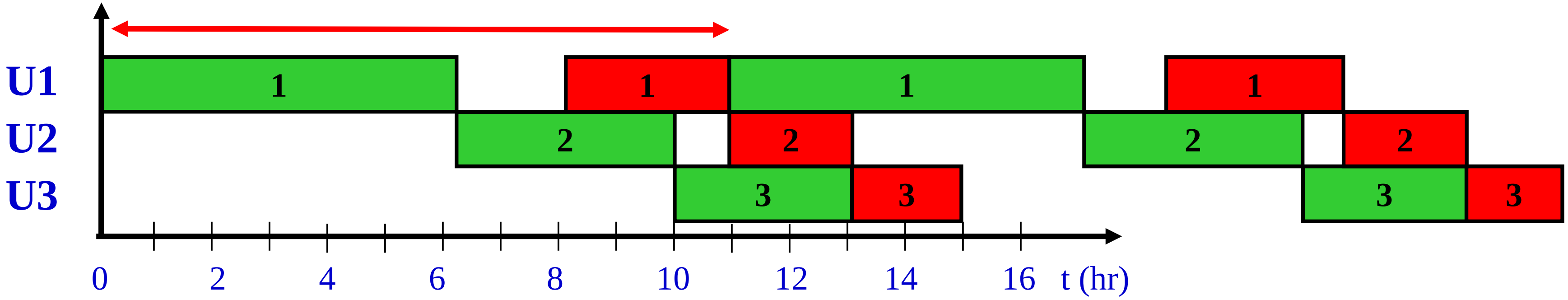
$n_{i,}$ # of lots of product i M, # of stages

Example

Product	Stage 1	Stage 2	stage 3
A	6	4	3
B	3	2	2

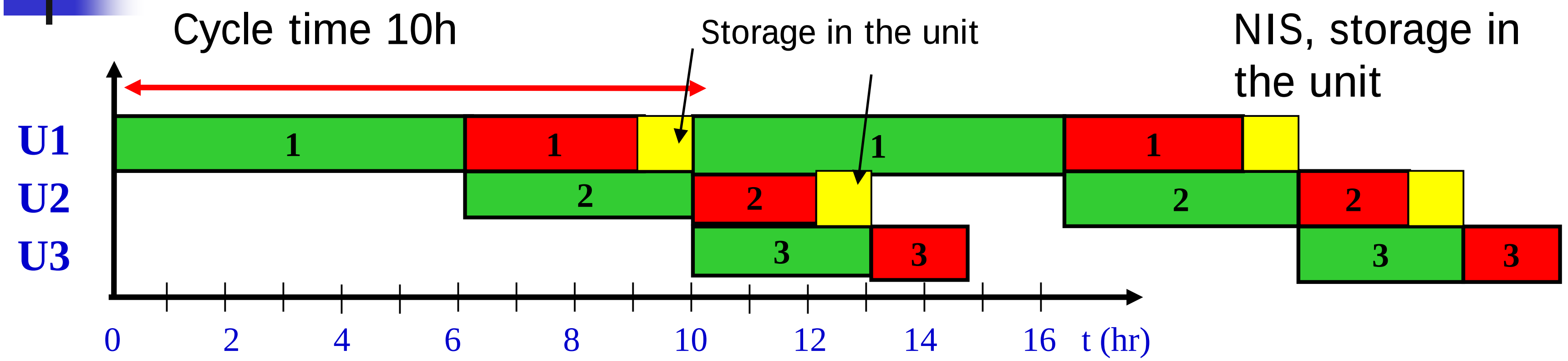
Campaign ABAB

Cycle time 11h

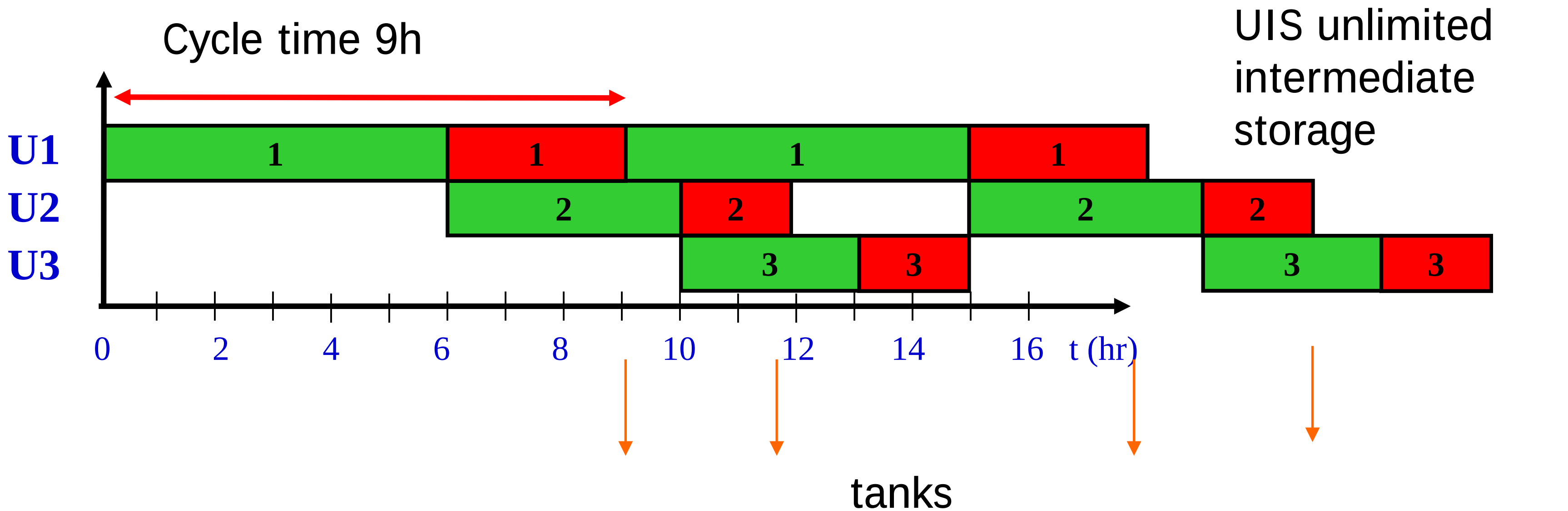


ZW Zero wait transfer

Example

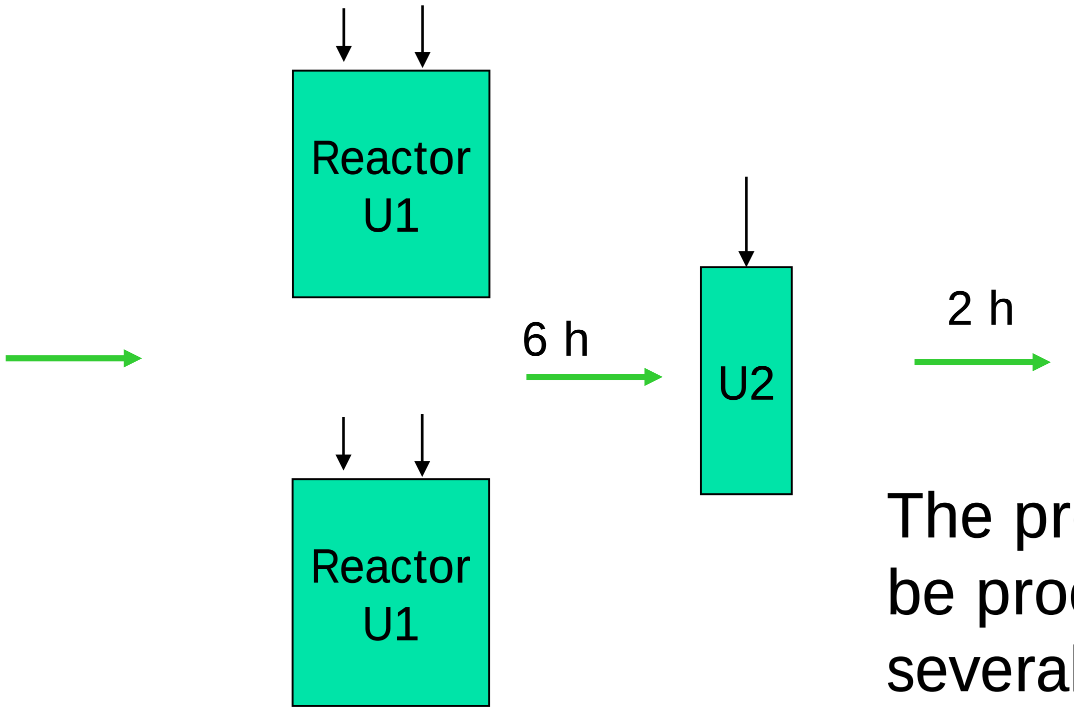


Cycle time 10h
Storage in the unit
NIS, storage in the unit
U1
U2
U3
1
1
1
1
2
2
2
2
3
3
3
3
0
2
4
6
8
10
12
14
16
t (hr)



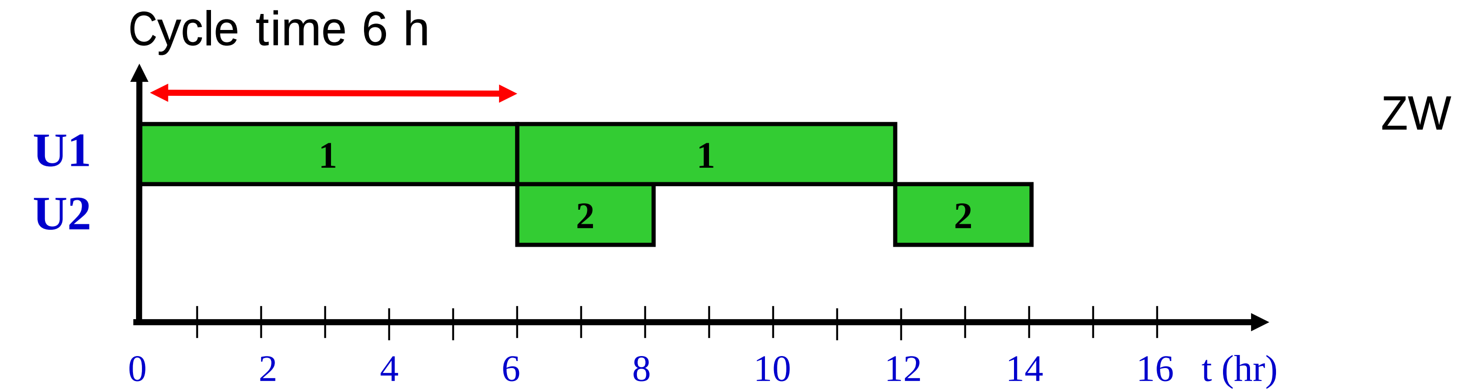
Cycle time 9h
UIS unlimited intermediate storage
U1
U2
U3
1
1
1
1
2
2
2
2
3
3
3
3
0
2
4
6
8
10
12
14
16
t (hr)
tanks

Parallel units

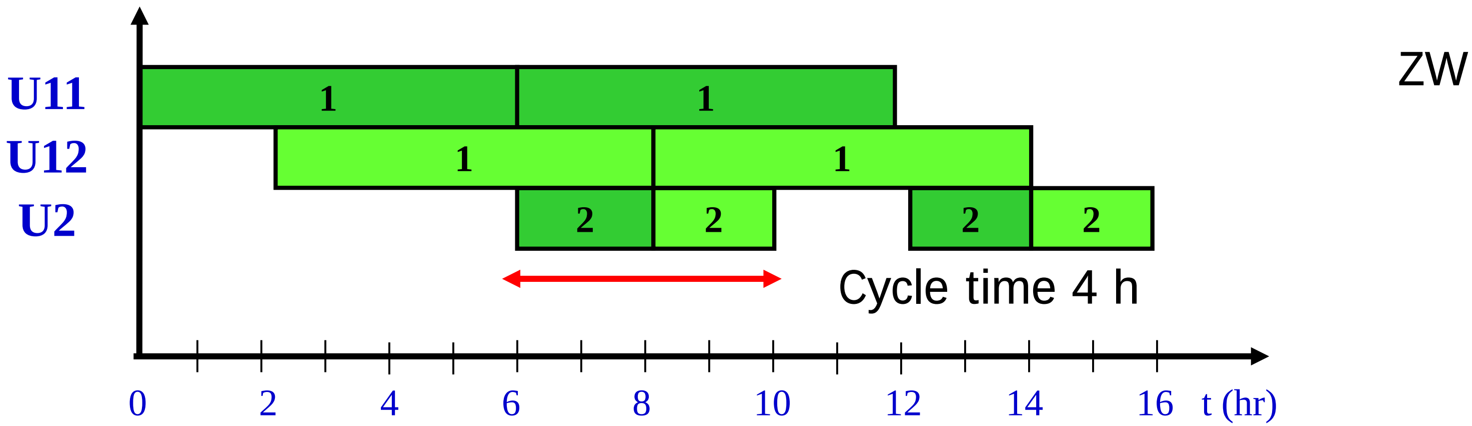


The products can be processed in several parallel units in some stages

Parallel units

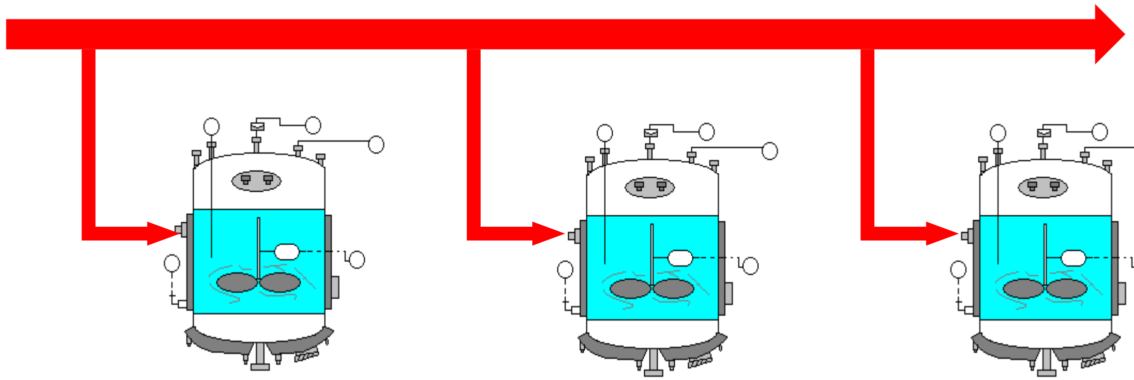


Without parallel units



With two parallel units in stage 1

Shared resources

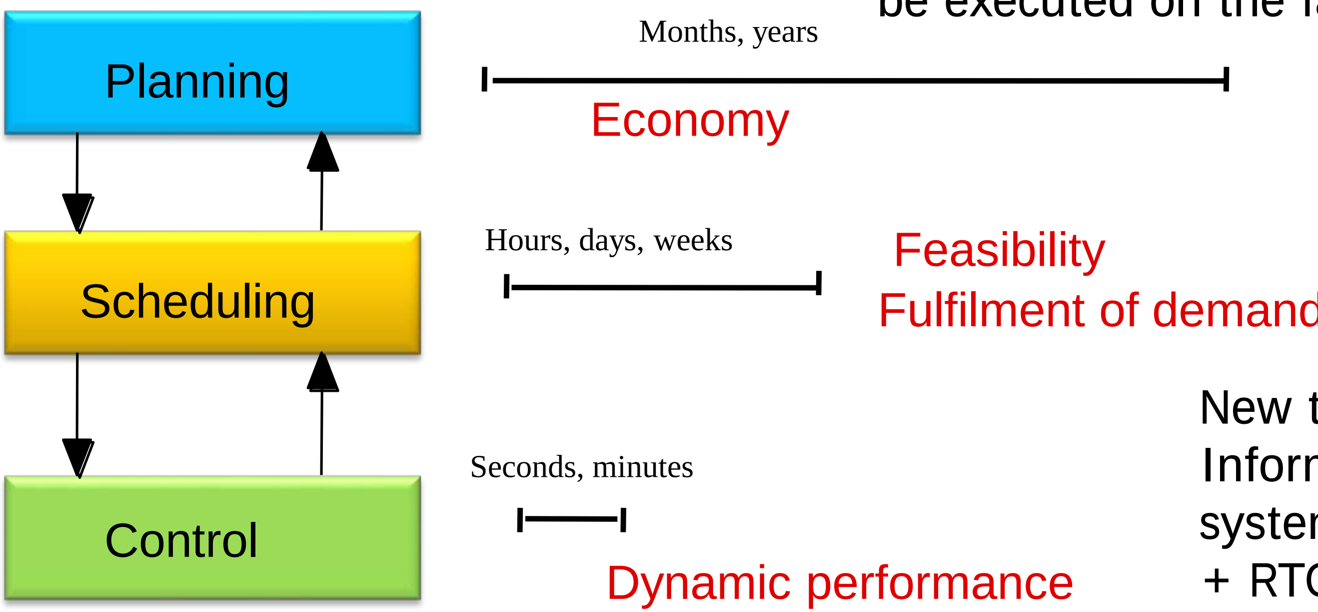


Processing tasks require utilities such as steam, electricity, cooling water, etc. and manpower that are shared among the different process units.

Besides the efficient allocation of task to units to meet product demands, it is also necessary to consider that simultaneously executed tasks do not to utilize resources outside their availability limits

Planning and scheduling

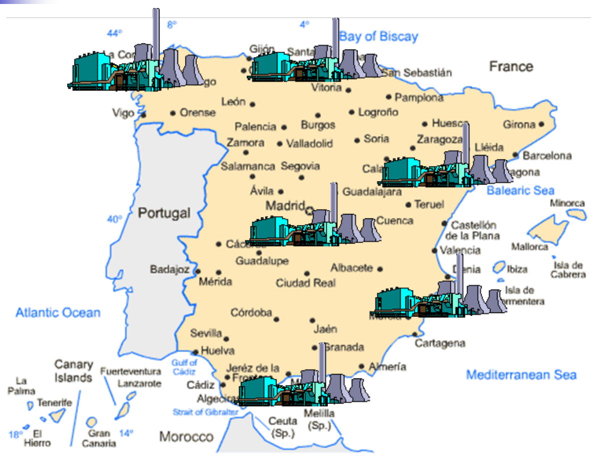
Scheduling translates the economic plan into a sequence of actions to be executed on the factory



New tools: Information systems, ERP, MES + RTO

Hierarchical decision making with different time scales, models and incertitude

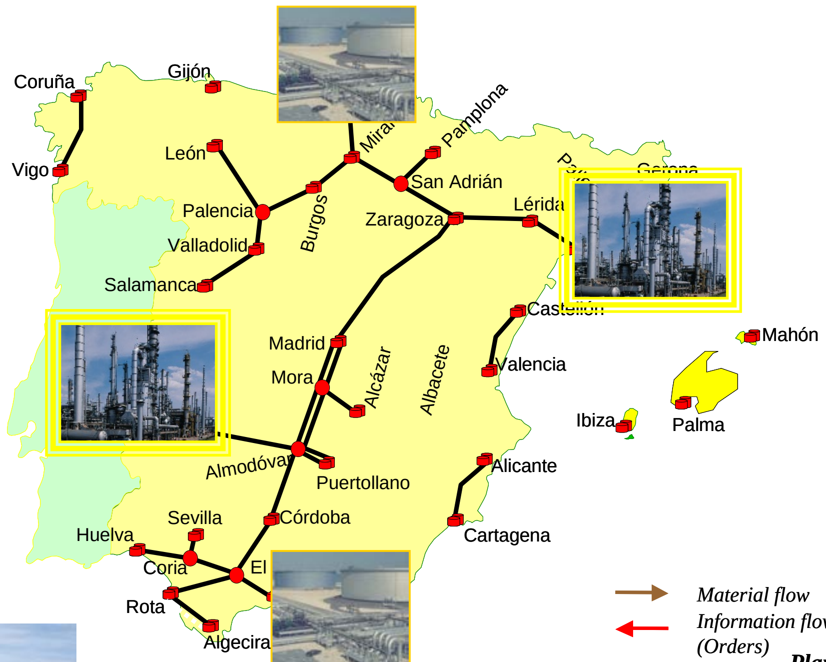
Planning and Scheduling



Planning: Allocate production of different products to different facilities in each time period over a medium-term horizon to fulfil customer demand, taking into account capacity constraints, inventory and transportation costs with the aim of minimizing total cost.

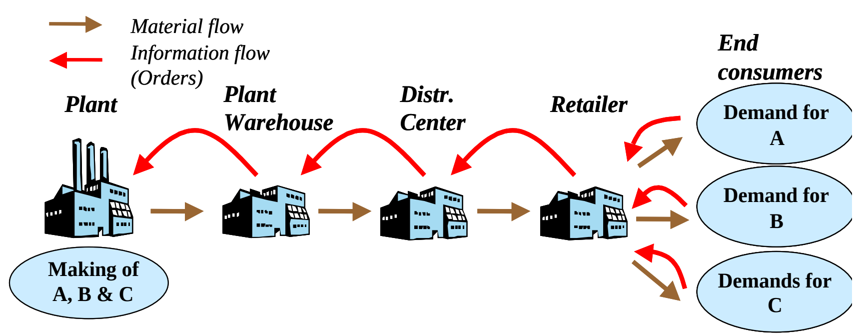
Scheduling: Allocate resources (equipment, utilities, people) to competing tasks and the sequencing of tasks to units of a single facility over a short-term horizon, using more detailed information with the aim of minimizing makespan, tardiness,...fulfilling production targets and constraints.

Logistics



Logistics: How to store, transport and distribute goods and resources over time to satisfy customers demands and supply constraints minimizing costs

Supply chains: a network between a company and its suppliers to produce and distribute a specific product to the final consumer

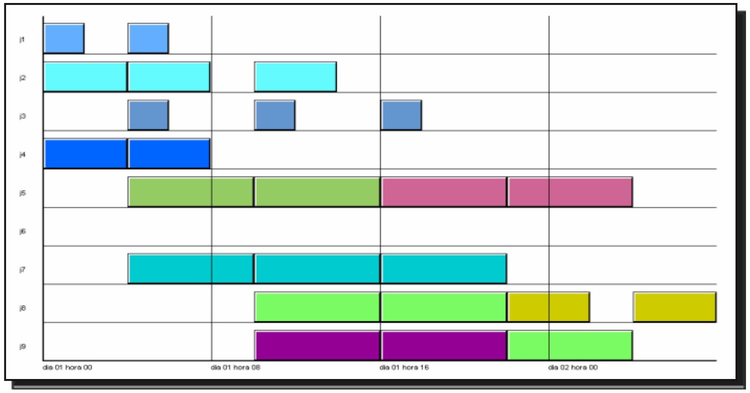


Planning and scheduling

- Planning and Scheduling
  - Strong industrial interest
  - Room for optimization
  - Components: Resources (equipment, utilities, people, materials,…), tasks (reactions, packing, cleaning, transportation,...) and time
- Problems:
  - How to formulate the operation of the system as an optimization problem including logic and constraints
  - How to solve efficiently the optimization problem
  - How to interpret and implement the solution

Mathematical Programming

- Main decision variables (real or binary):
  - Resources (units, utilities, people,...) to execute tasks at certain times
  - Amount of materials processed in each task
  - Inventory levels of materials over time
  - Sequence of tasks
  - Timing of tasks
  - ....



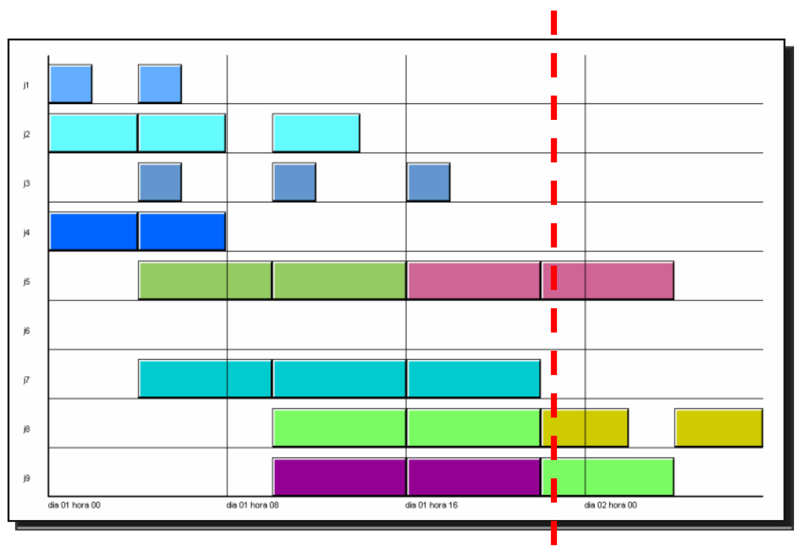
Gantt diagram

Mathematical Programming

- Main constraints:
  - Activities must proceed until completion
  - Resources cannot exceed its availability
  - Material balances
  - Processing or storage capacity
  - Satisfaction of order by its due date
  - ....

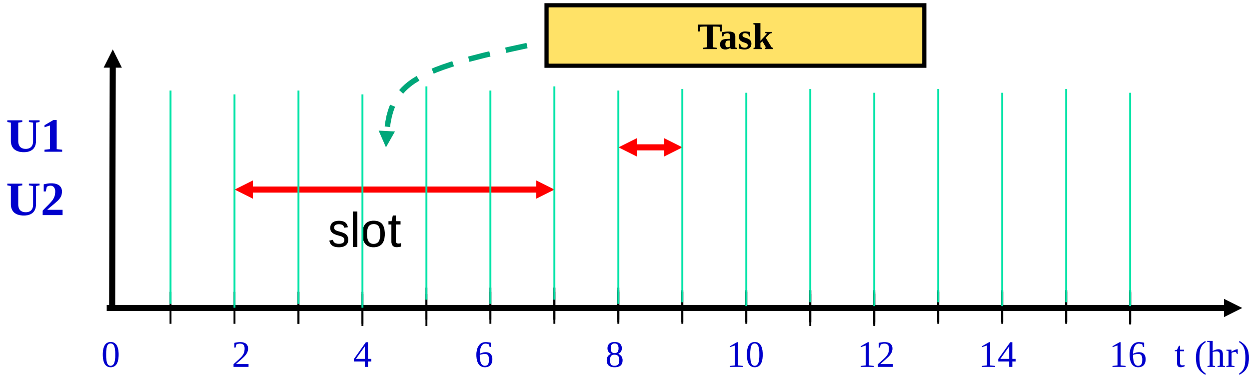
Mathematical Programming

- Typical aims:
  - time required to complete all tasks (makespan)
  - number of tasks completed after their due dates
  - plant throughput
  - Tardiness , lateness
  - profit
  - costs

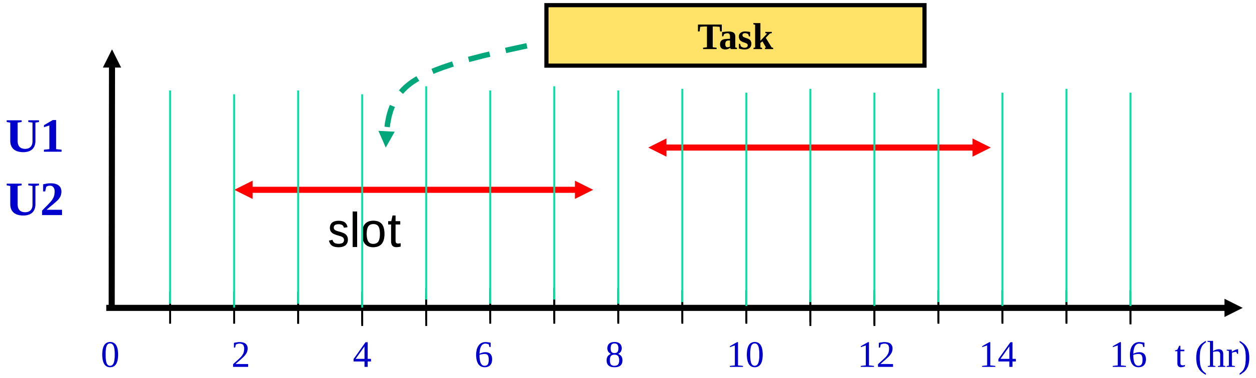


Time domain representation

Time slots: time intervals for allocation of tasks to units



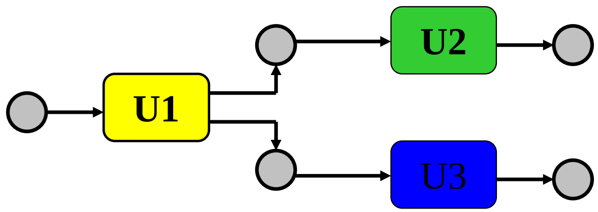
Discrete time: Slots start and end at points of a fix discrete time grid

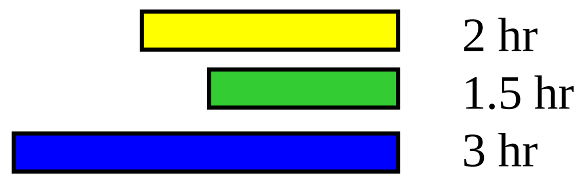


Continuous time: slots can be of any time length

Discrete time representation

Tasks



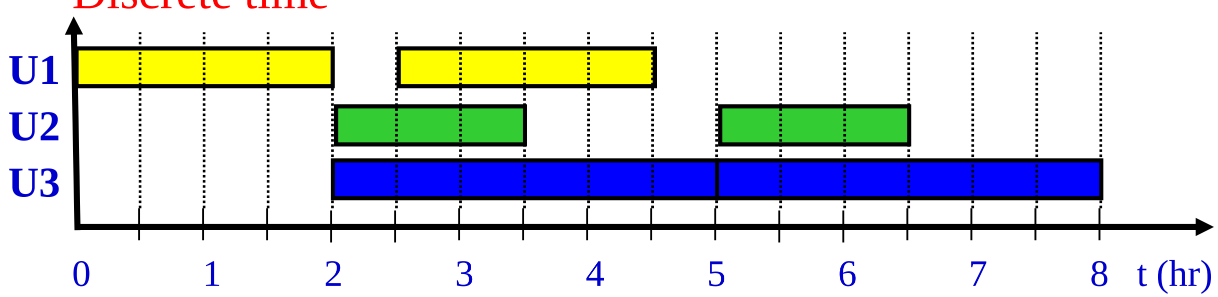


⇨ Interval = 0.5 hr

- Tasks are forced to last an integer number of discretization intervals

- Fix number of regular intervals of time.
- Events can only take place at these times.
- Balance between problem size (number of intervals) and approximation to reality
- Easier to formulate shared constraints

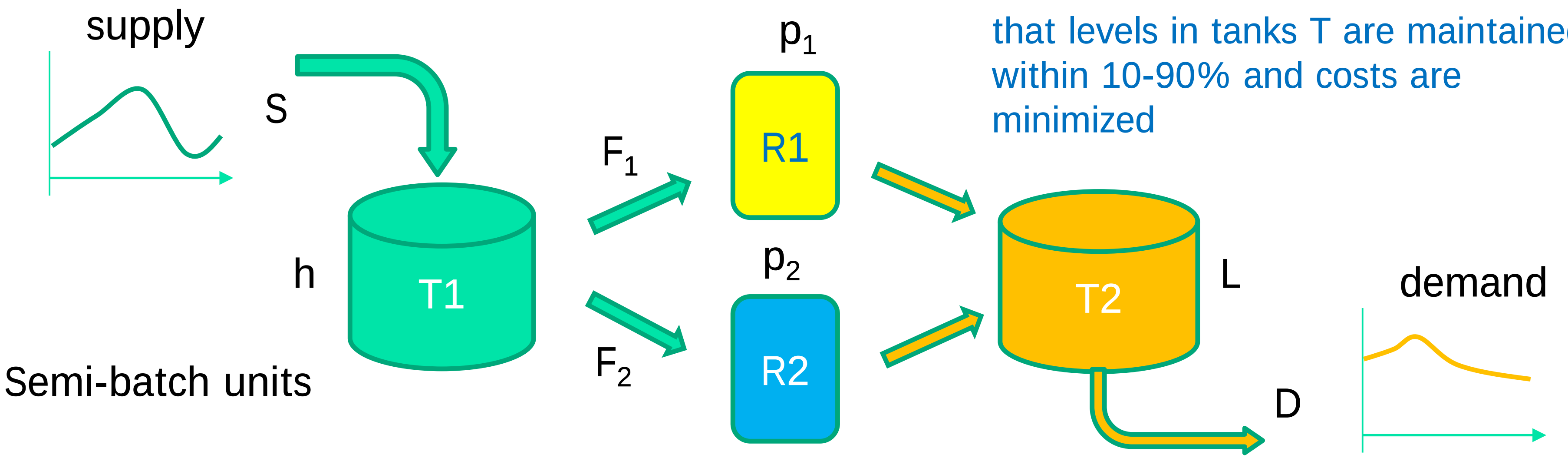
Discrete time



Integer variables y_{ij} are used to represent if task i operates in slot j

Discrete time example

Schedule the semi-batch units R so that levels in tanks T are maintained within 10-90% and costs are minimized

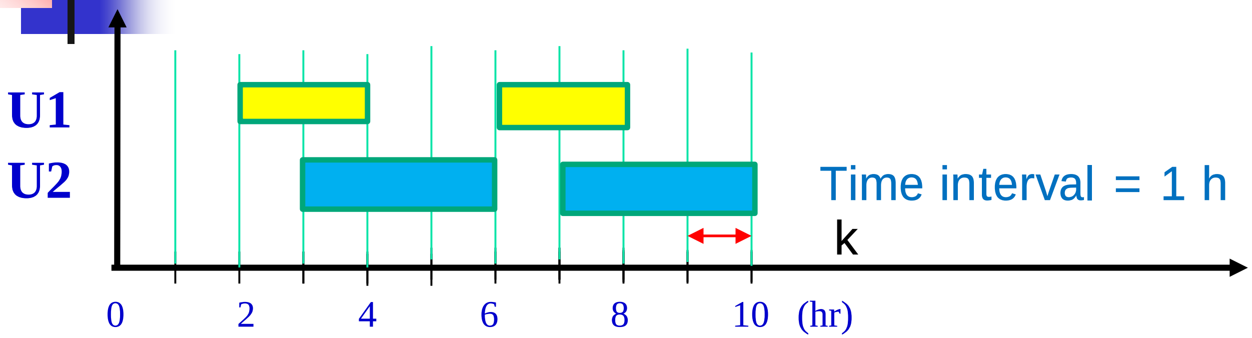


	p Batch processing time(h)	F inflow to batch units in operation (m3/h)	C Cost (€) per hour of processing	Tank volume (m3)	Initial tank volume (m3)	Tank cross section (m2)
Reactor 1	2	3.5	100			
Reactor 2	3	2	140			
Tank 1				20	10	4
Tank 2				22.5	12	4.5

Expected supply and demand may change every hour according to a table

Time (h):	1	2	3	4	5	6	7	8	9	10
Supply (m3/h)	5	5	2	6	3	5	4	3	1	0
Demand (m3/h)	2	1	5	6	7	2	2	6	4	0

Mathematical formulation



$$\min_{y,ys,yd,h,n} \sum_{i=1}^{2}\sum_{k=1}^{10} C_i y_{ik}$$

$$4h_{k+1} = 4h_k + S_k - \sum_{i=1}^{2} F_i y_{ik} \qquad \forall k$$

$$4n_{k+1} = 4n_k - D_k + \sum_{i=1}^{2} p_i F_i yd_{ik} \qquad \forall k$$

$$p_i ys_{ik} \le \sum_{j=0}^{p_i-1} y_{i,k+j} \qquad \forall i \quad \forall k \qquad ys_{i,k-p_i} = yd_{i,k} \qquad \forall i \quad \forall k$$

$$(p_i - 1)(1 - ys_{ik}) \ge \sum_{j=1}^{p_i-1} ys_{i,k+j} \qquad \forall i \quad \forall k$$

ys_{ik} = 1 if unit i starts at time k binary

yd_{ik} = 1 if unit i descharges at time k binary

y_{ik} = 1 if unit i is operating at time k binary

F_i flow to unit i in operation

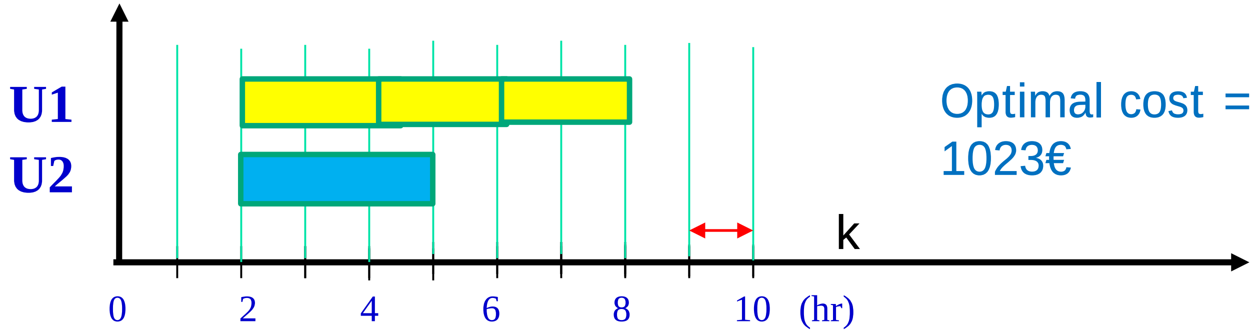
p_i batch cycle of unit i

h_k level of supply tank at time k

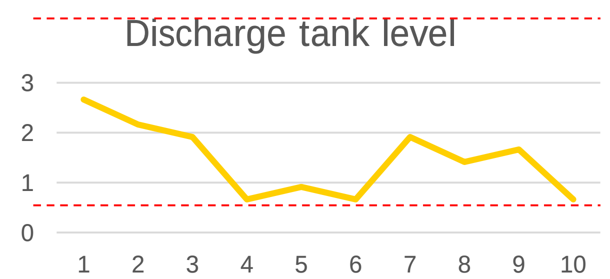
n_k level of discharge tank at time k

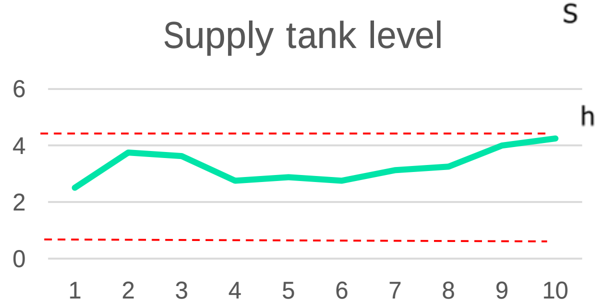
C_i cost per hour of operation of unit i

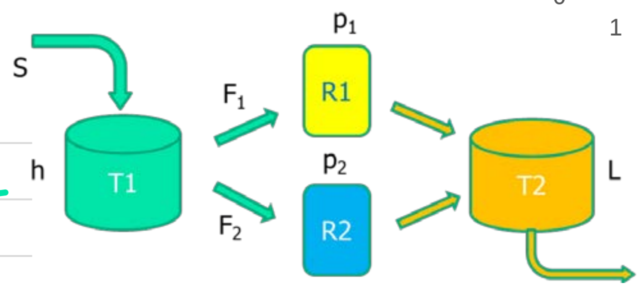
Solution

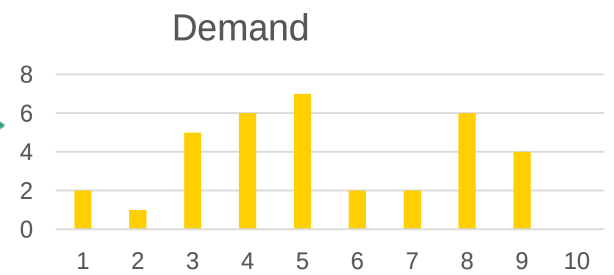


Optimal cost = 1023€

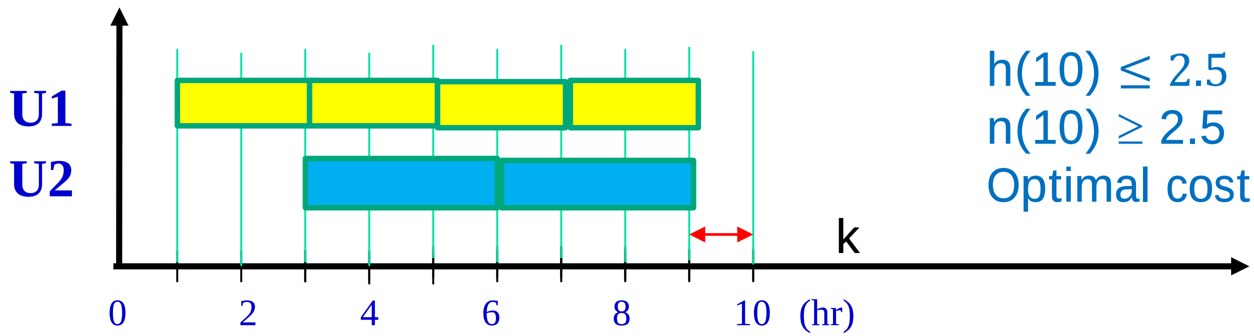




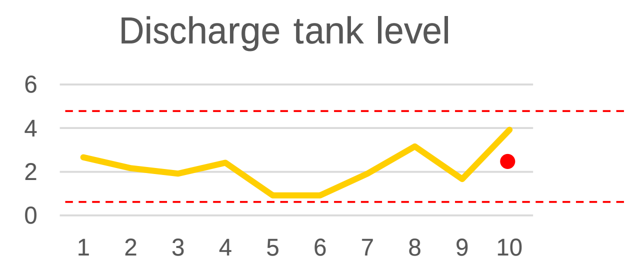


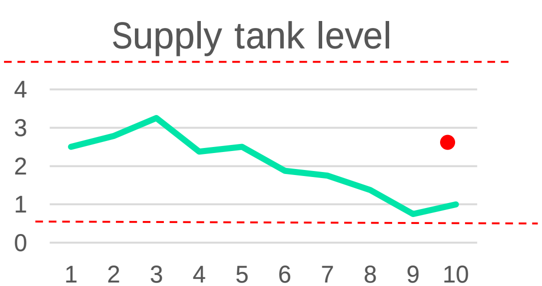


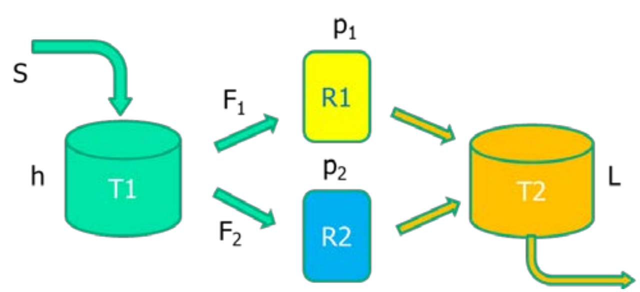
Solution with terminal constraints

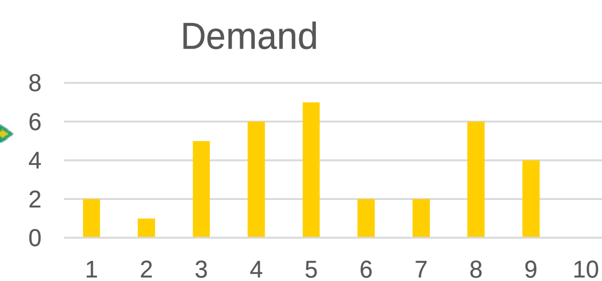


$h(10) \leq 2.5$
$n(10) \geq 2.5$
Optimal cost = 1644€

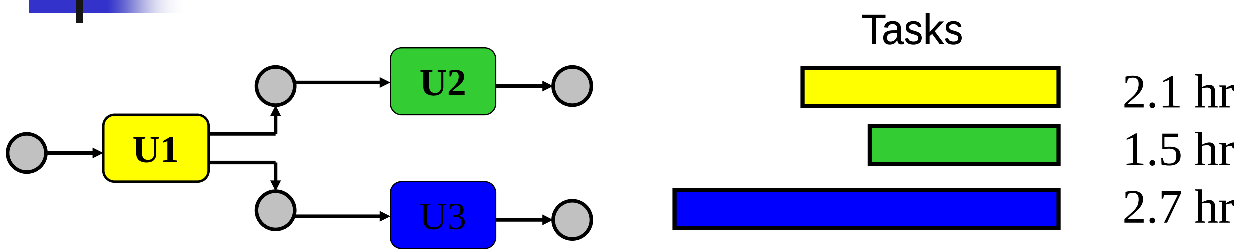




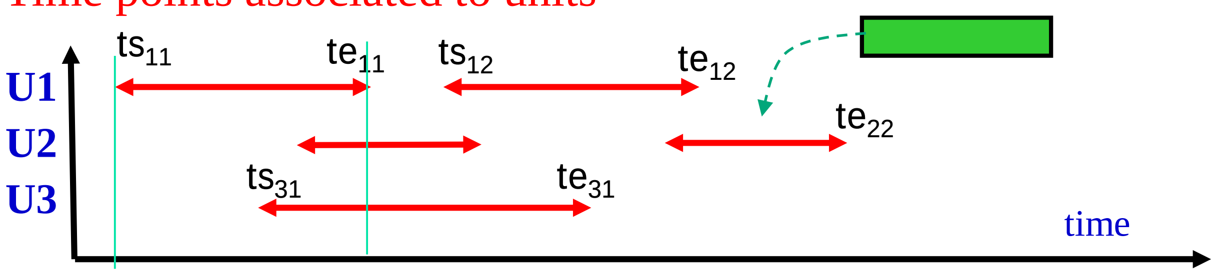


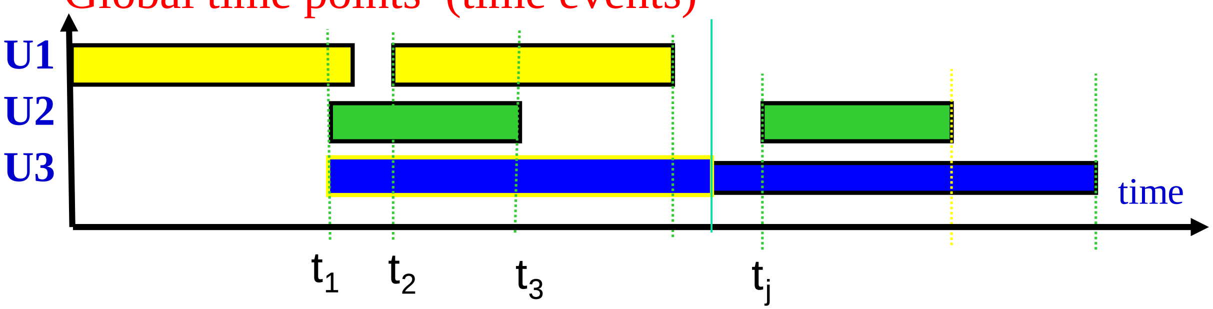


Continuous time representation



- Tasks may last any duration





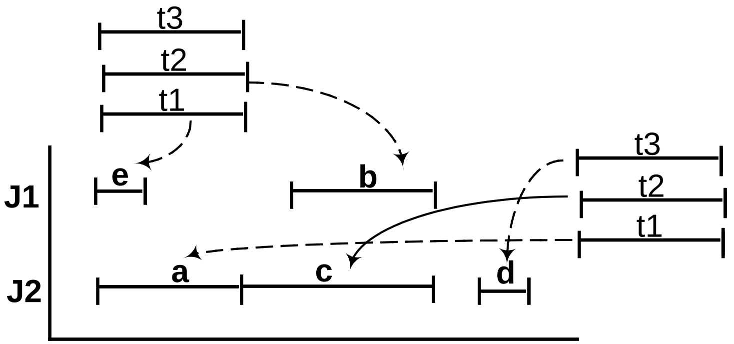
- Tasks linked to slots. Slots start or stop at any time.
- Number of time slots have to be defined previously
- Smaller number of time variables
- More difficult to deal with shared constraints
- Time instants t are new variables of the scheduling problem
- Events t_j can take place at any time. Its number has to be defined previously

Event representation

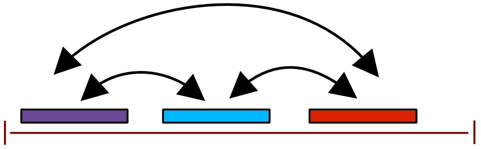
For flowshop problems

Time slots: Tasks must be assigned to each time slot

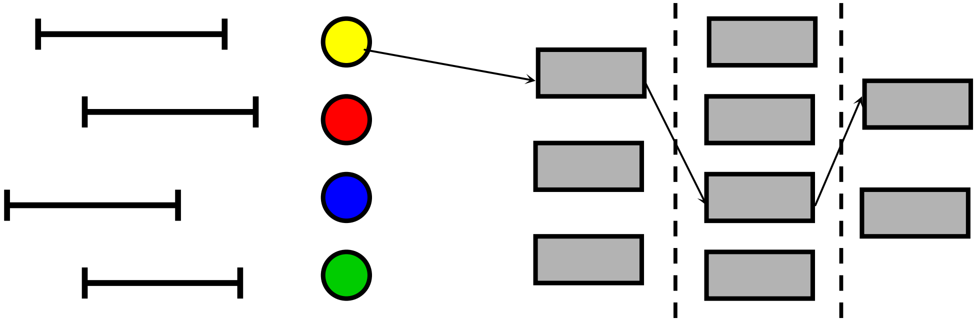
Predefined number of time points or slots
How many?



General or inmediate precedence to order tasks over time

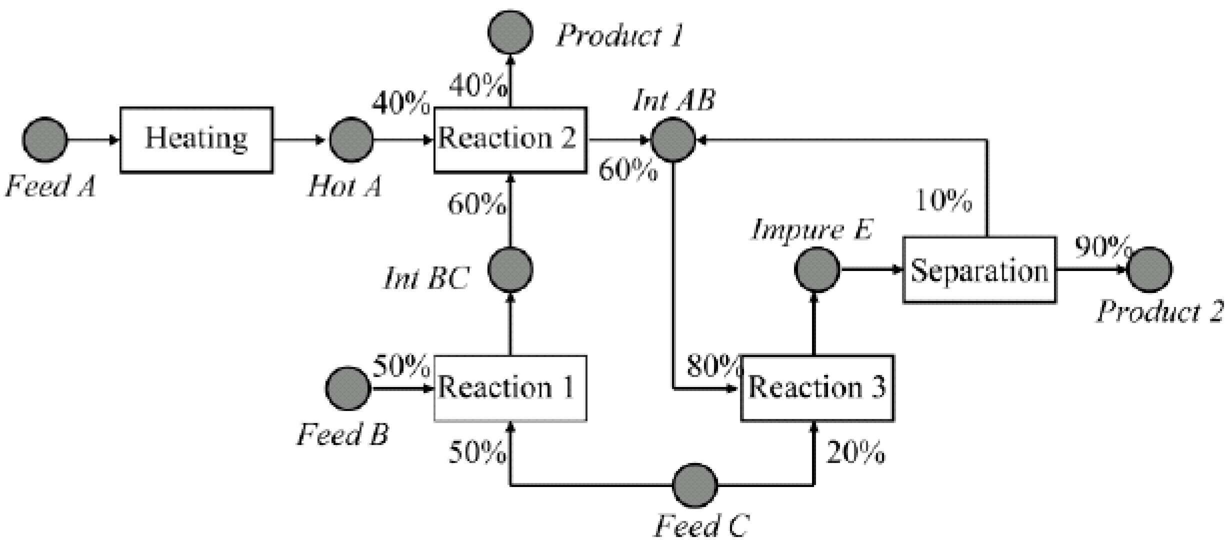


Two main types of problems



Flowshop

Allocation, sequencing, routing
No recycling or splitting,mixing

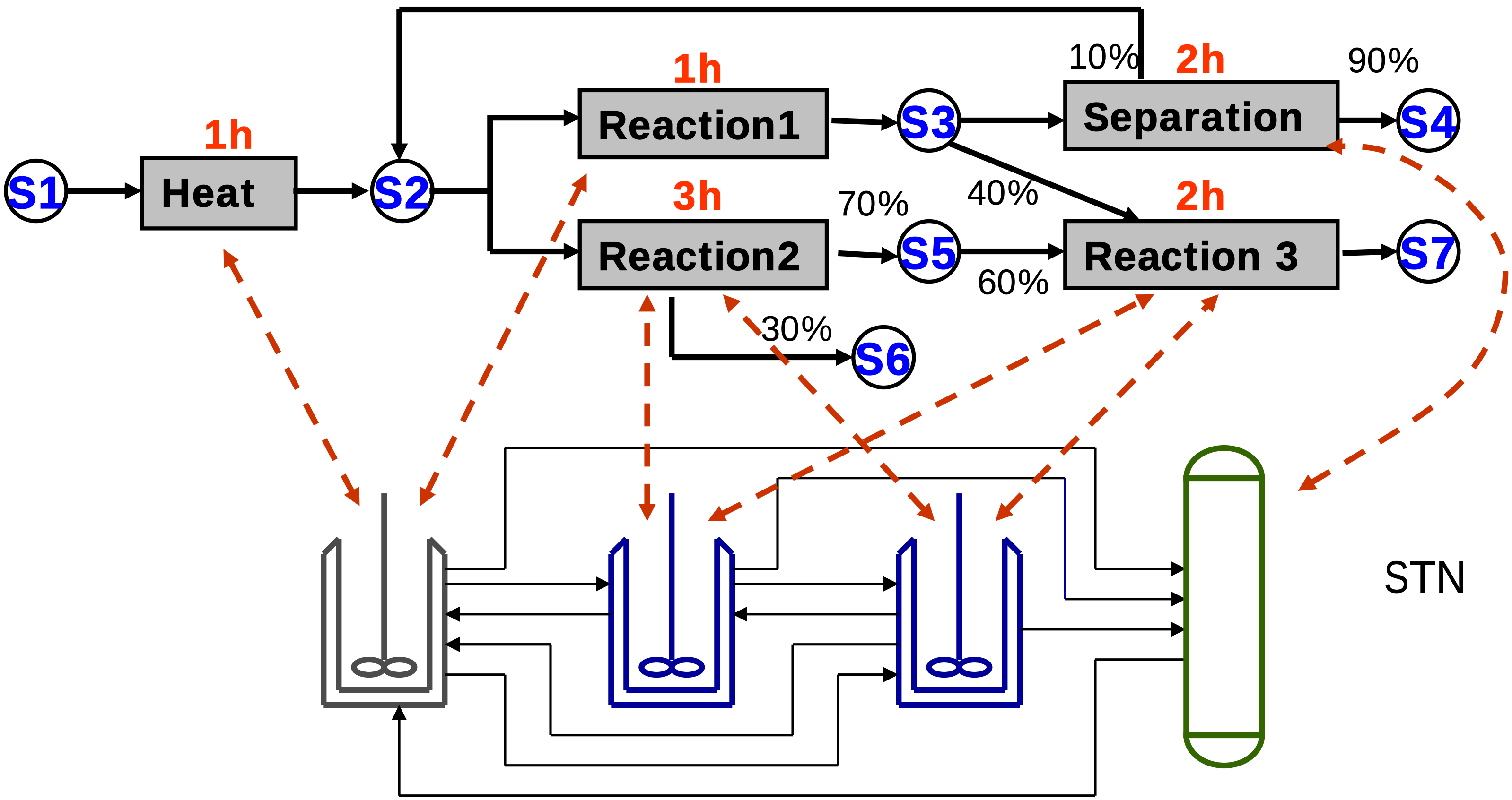


Networks

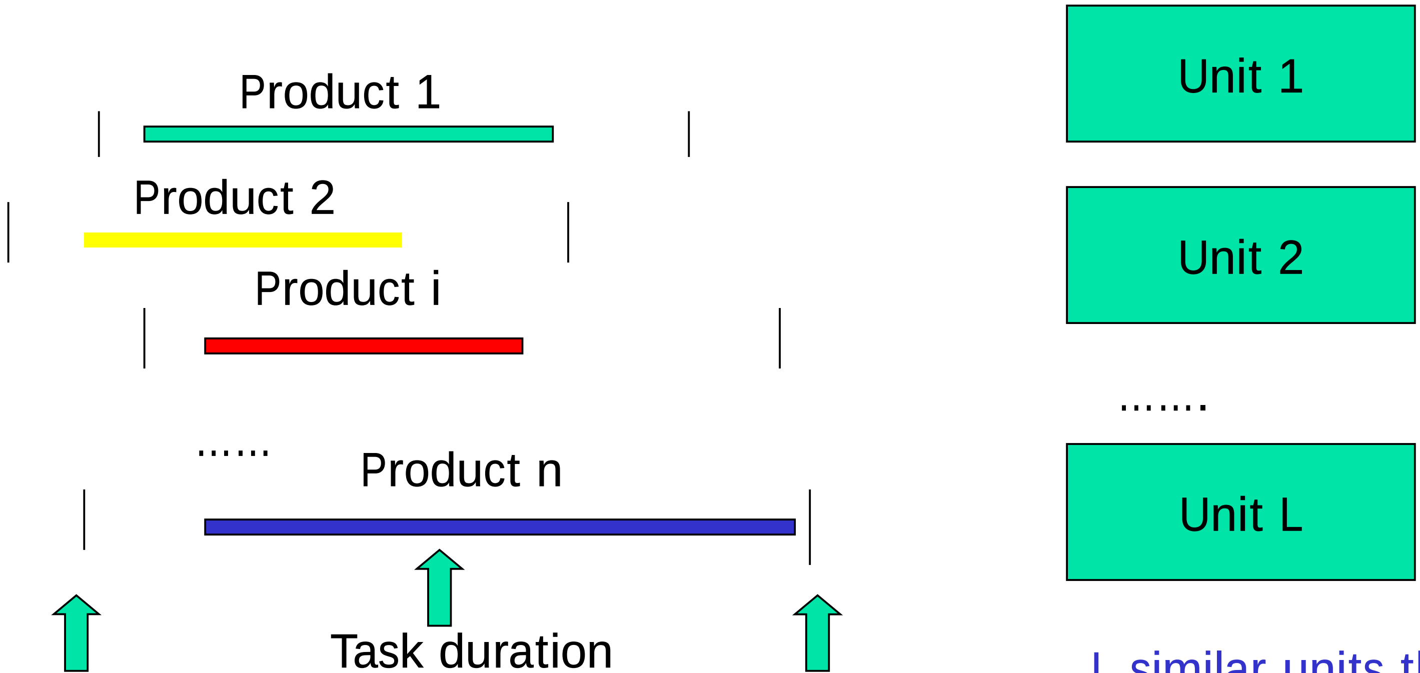
STN

RTN

Network problems



Multiproduct single stage scheduling



Minimum starting time (Release time)

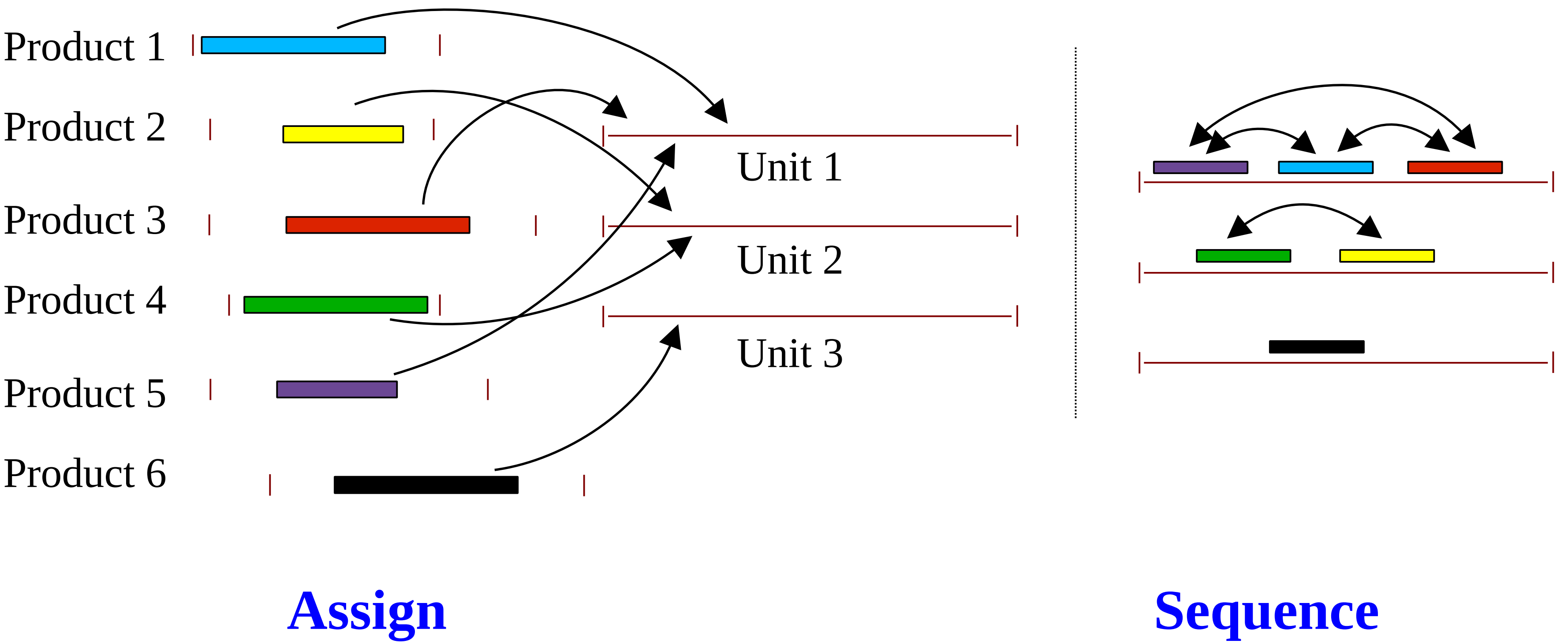
Maximum ending time (Due date)

L similar units that can perform the task, but, perhaps with different costs or processing times

Each of the n products must be processed in a unit and only in one

Assigning and sequencing

Assign each product to a unit and compute processing order and execution time, so that the time constraints are satisfied and the processing costs are minimized



Allocation MILP

$$y_{im} = \begin{cases} 1 \text{ if product } i \text{ is assigned to unit } m \\ 0 \text{ otherwise} \end{cases}$$

$$\min_{y,ts} \sum_{i \in I} \sum_{m \in M} C_{im} y_{im}$$

Cost

$$\text{s.t.} \quad ts_i \geq r_i$$

Start time t_i

$$ts_i + \sum_{m \in M} p_{im} y_{im} \leq d_i \quad \forall i \in I$$

$$\sum_{m \in M} y_{im} = 1 \quad \forall i \in I$$

Unit assigment

$$\sum_{i \in I} y_{im} p_{im} \leq \max_i \{d_i\} - \min_i \{r_i\} \quad \forall m \in M$$

Total processing time in a unit m

r_i minimum starting time of product i

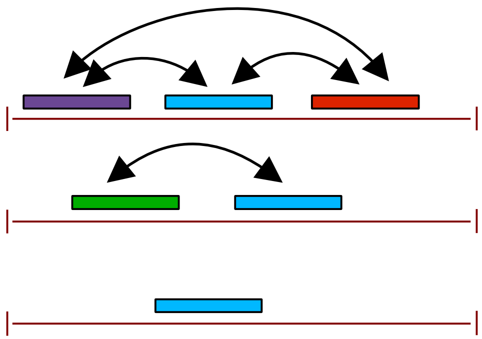
d_i maximum ending time of product i

p_{im} processing time of product i in unit m

C_{im} cost of processing product i in unit m

ts_i starting time of product i

Sequencing within every unit



$$z_{ij} = \begin{cases} 1 & \text{If product i precedes product j in unit m} \\ 0 & \text{Otherwise} \end{cases}$$

$$y_{im} = \begin{cases} 1 & \text{If product i is assigned to unit m} \\ 0 & \text{Otherwise} \end{cases}$$

If product i and product j are assigned to unit m, then i precedes j or j precedes i

$$1 \geq z_{ij} + z_{ji} \geq y_{im} + y_{jm} - 1 \quad \forall i, j \in I,\ i > j,\ m \in M$$

If product i precedes product j in unit m, then the start time of product j must be larger than the start time of product i plus task i duration

$$ts_j \geq ts_i + \sum_{m \in M} p_{im} y_{im} - M(1 - z_{ij}) \quad \forall i, j \in I,\ i \neq j$$

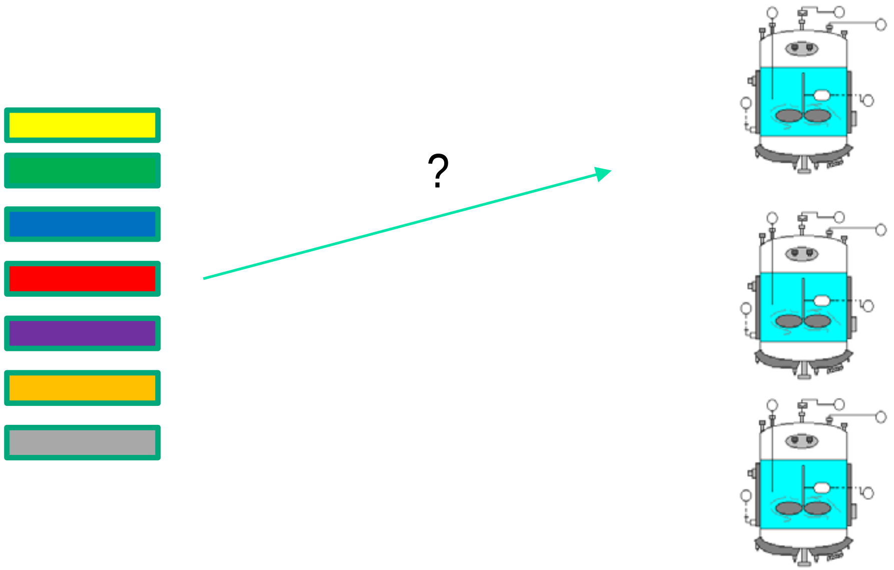
Big-M Constraint

$$y_{im} = \{0,1\},\ z_{ij} = \{0,1\}, \qquad ts_i \geq 0$$

MILP problem

Optimal assignment of chemical products to batch reactors (single stage multiproduct , 7 products and 3 reactors).

Product	Minimum starting time	Due date	Cost in reactor 1	Cost in reactor 2	Cost in reactor 3	Processing time in reactor 1	Processing time in reactor 2	Processing time in reactor 3
A	1	8	3	2	4	2	3	2
B	1	10	5	6	5	3	3	2
C	4	7	4	3	5	1	1	1
D	2	9	5	2	7	2	3	2
E	3	7	3	2	6	3	2	3
F	1	7	3	3	5	3	4	4
G	3	8	4	4	6	1	1	1



Find the assignment of products to reactors, and its sequencing, that fulfils time constraints and optimize processing costs

Optimal cost solution

Product	Minimum starting time	Due date	Cost in reactor 1	Cost in reactor 2	Cost in reactor 3	Processing time in reactor 1	Processing time in reactor 2	Processing time in reactor 3
A	1	8	3	2	4	2	3	2
B	1	10	5	6	5	3	3	2
C	4	7	4	3	5	1	1	1
D	2	9	5	2	7	2	3	2
E	3	7	3	2	6	3	2	3
F	1	7	3	3	5	3	4	4
G	3	8	4	4	6	1	1	1

Optimal cost: 22

Reactor 1	F			G	A		B		
Reactor 2			E		C	D			
Reactor3									
	1	2	3	4	5	6	7	8	9

Gantt diagram

Related problems

$$\min_{y_{im},W} \quad W$$

$$ts_i + \sum_m y_{im} p_{im} \le W \qquad i \in I$$

$$0 \le \quad W \le \quad W_{max}$$

Different aim: minimize makespan W

Makespan is an upper bound for the end time of operation of each task

$$\sum_{m \in F_i} y_{im} = 1 \qquad i \in I$$

$$y_{im} = 0 \quad m \notin F_i$$

Each task i can be performed only in a subset F_i of M

$$y_{i_1 m} + y_{i_2 m} \le 1 \qquad i_1, i_2 \in N$$

$$y_{im} \in \{0,1\}$$

Tasks 1_1 and i_2 cannot be performed in the same equipment m

Minimum makespan problem

Product	Minimum starting time	Due date	Cost in reactor 1	Cost in reactor 2	Cost in reactor 3	Processing time in reactor 1	Processing time in reactor 2	Processing time in reactor 3
A	1	8	3	2	4	2	3	2
B	1	10	5	6	5	3	3	2
C	4	7	4	3	5	1	1	1
D	2	9	5	2	7	2	3	2
E	3	7	3	2	6	3	2	3
F	1	7	3	3	5	3	4	4
G	3	8	4	4	6	1	1	1

Optimal makespan: 6 h

Reactor 1	A		D		G				
Reactor 2		F			C				
Reactor3		B		E					
	1	2	3	4	5	6	7	8	9

Gantt diagram