PFC
Predictive Functional Control

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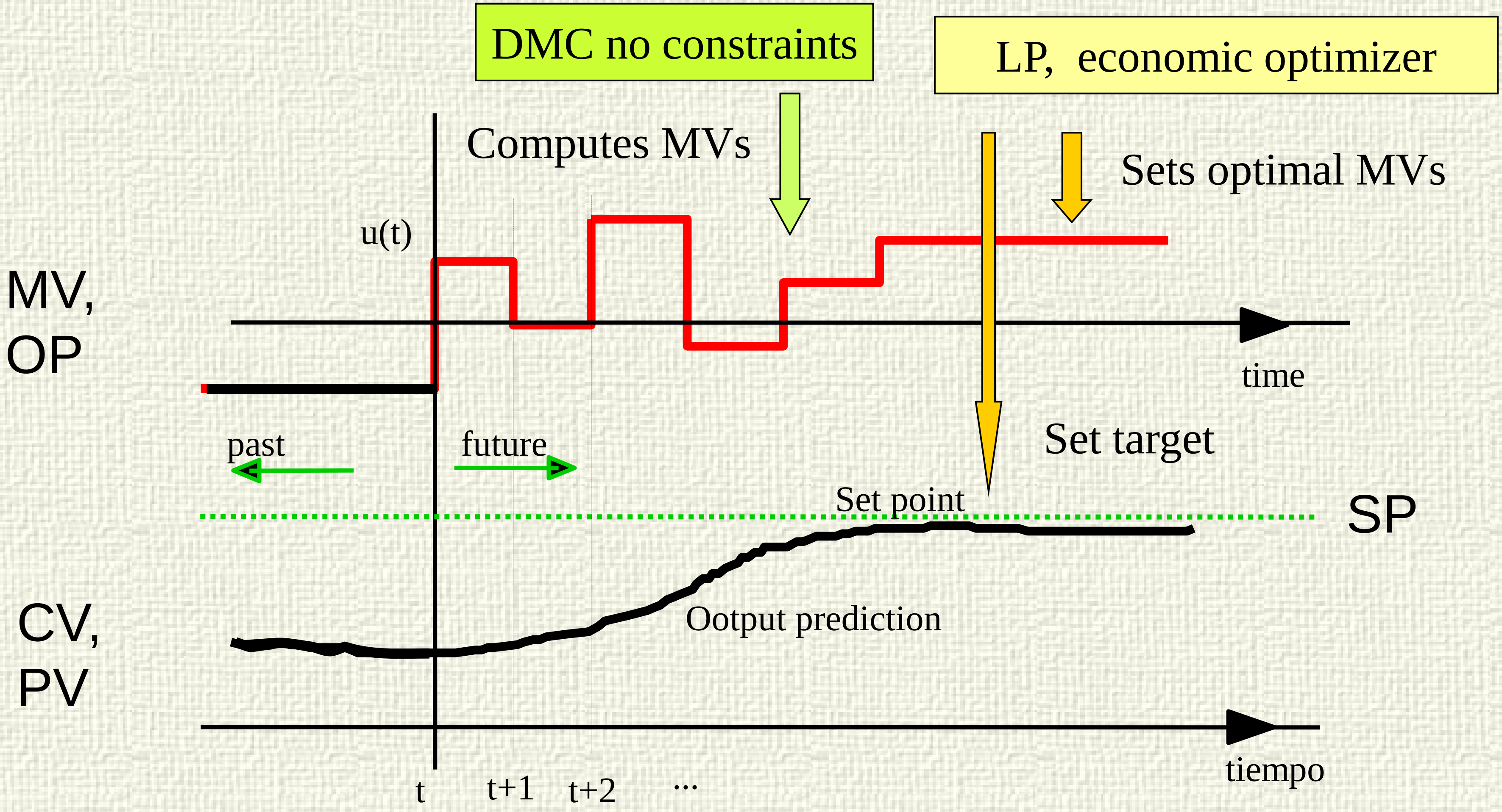
Outline

- ✓ As simple as possible
  - – Motivation
  - – PFC main ideas
  - – An introductory example

Motivation

- Predictive control is a widely used control technique in the process industry combined with an optimization layer
- Most of the implementations use internal linear models (step responses) and LP optimizers

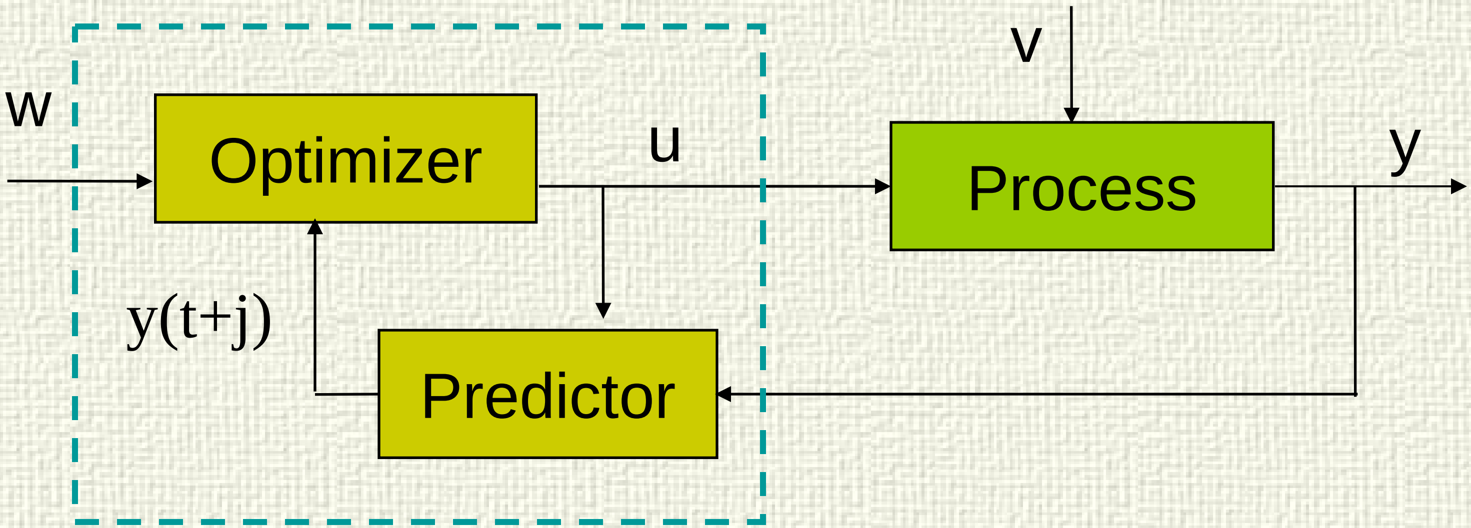
DMC


DMC no constraints
LP, economic optimizer
Computes MVs
Sets optimal MVs
u(t)
MV,
OP
time
past
future
Set target
Set point
SP
Ootput prediction
CV,
PV
tiempo
t
t+1
t+2
...

Motivation

- When a significant non-linear process is faced, a non-linear controller based on a non-linear plant model is required.
- Current approaches based on first-principles models, NN, volterra series, etc. lead to a non-linear optimization problem that must be solved on-line
- This is a heavy burden, both from the implementation and computational load

Non-linear Predictive Control

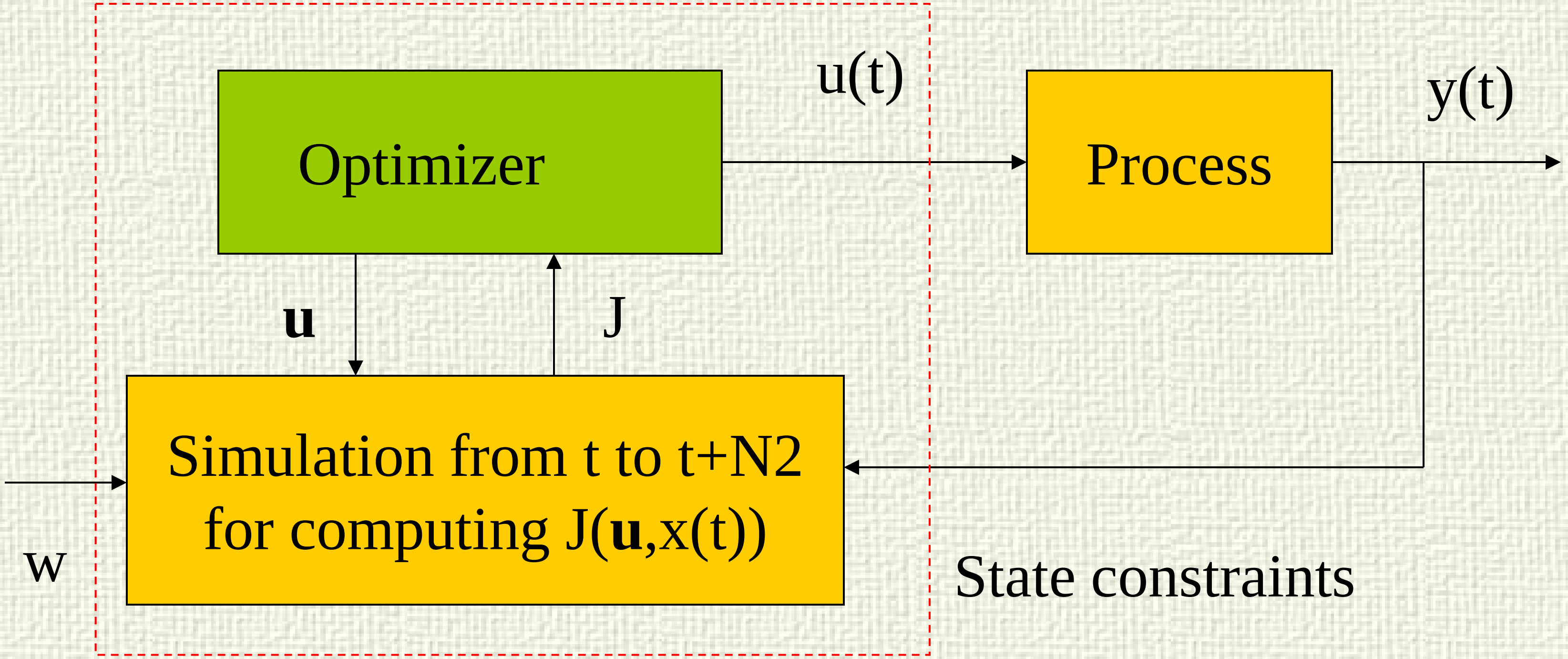


$$\min_{\Delta u} J = \sum_{j=N1}^{N2} \left[\hat{y}(t+j) - w(t+j)\right]^2 + \sum_{j=0}^{Nu-1} \left[\beta \Delta u(t+j)\right]^2$$

$$\dot{x} = f(x(t), u(t)) \qquad y(t) = g(x(t), u(t))$$

$$\underline{y} \le y(t+j) \le \overline{y} \qquad \underline{u} \le u(t+j) \le \overline{u}$$

Solving the dynamic NLP problem



State constraints

Gradients

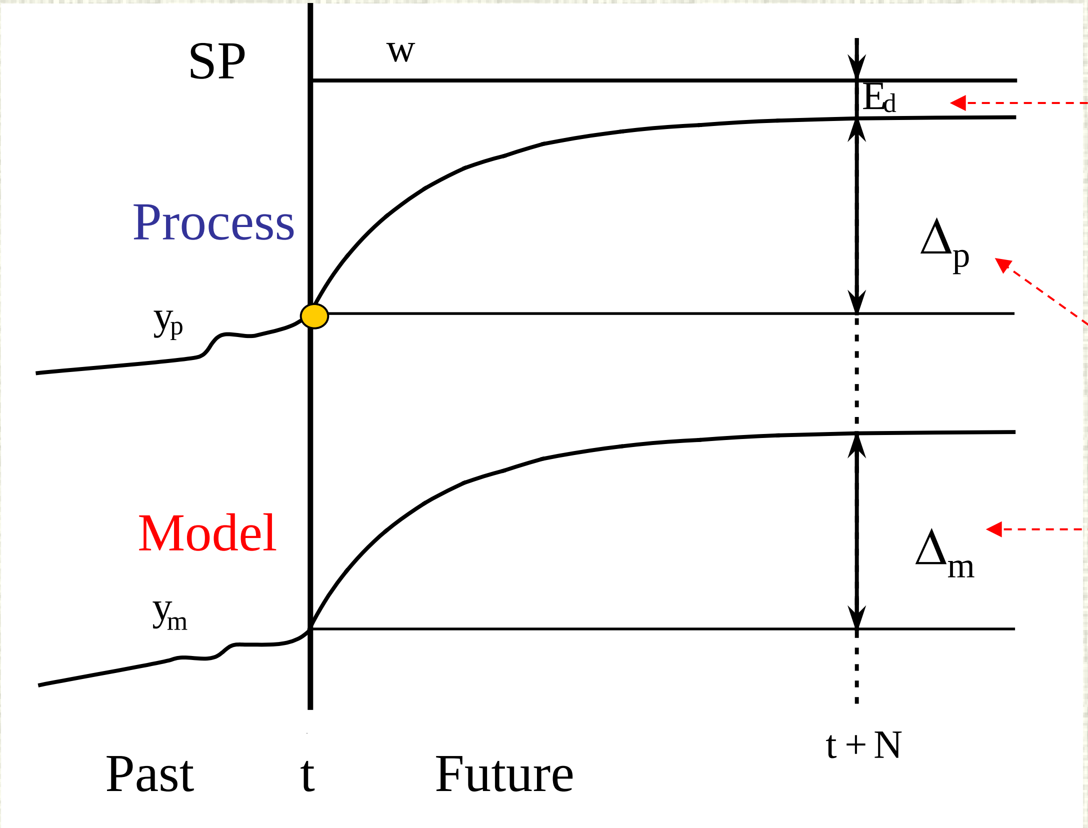
Aim

- The aim of Parametric Predictive Control (PPC) is to facilitate the implementation of the controller while retaining the main non-linear characteristics in its internal model
- Good compromise between speed of execution, easy of implementation and performance
- Target: Embedded controller in a DCS

PPC Main ideas

- Combines first principles models with MPC ideas and generates a simplified solution that is updated every sampling time.
- *Parametric Predictive Control (PPC)* (J. Richalet, 1996) was developed for, and successfully applied to, temperature control of batch reactors.
- A. Assandri, A. Rueda, PhD students
- Three steps:
  - Basic ideas presented in the linear case
  - Extended to the non-linear case with a chemical reactor CSTR
  - Industrial application to the bottom temperature control of a distillation column heated with a furnace-reboiler.

Basic concepts



Aim: Decrease the error in the future until a certain percentage of the current error w – y(t)

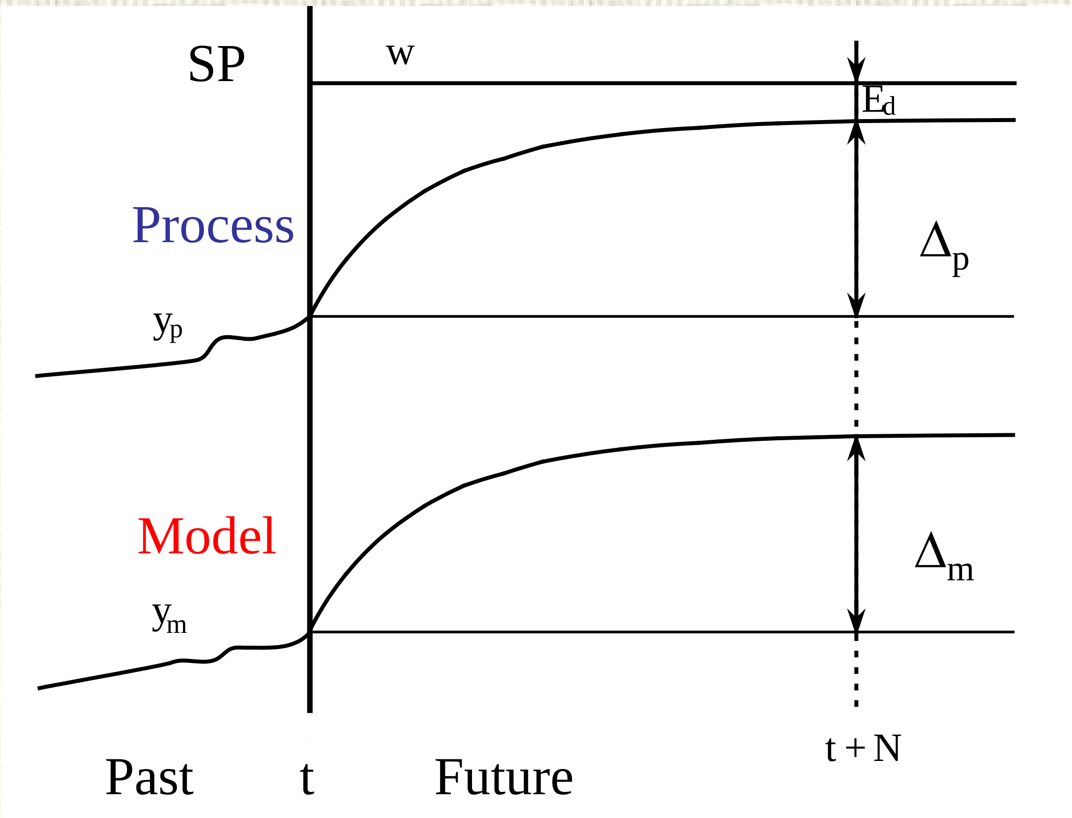
This implies to change the process output by Δ_p

Use the model to compute the control u(t) that provides a change in the model output Δ_m (u) = Δ_p

Key design equation: $\Delta_m (u) = \Delta_p$

Basic concepts

The future error at t+N must be a fraction of the current error w- y(t)



$$E_d = \lambda^N (w - y_p(t)) \qquad 0 < \lambda < 1$$

$$\Delta_p = (1 - \lambda^N)(w - y_p(t))$$

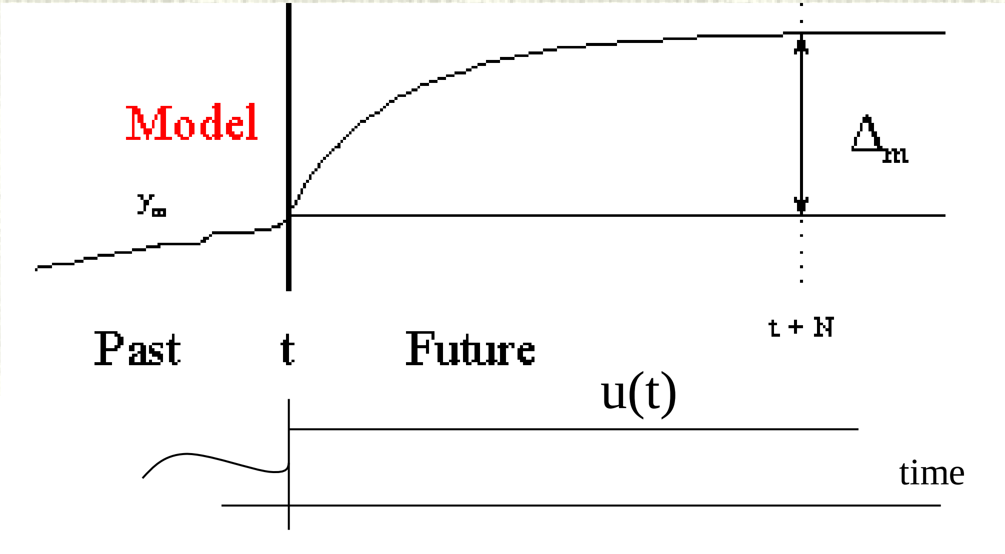
Example: First order model

$$\tau \frac{dy(t)}{dt} + y(t) = k\,u(t)$$

Basic concepts (Monoreg)

Example: First order model

$$\tau \frac{dy(t)}{dt} + y(t) = k\,u(t)$$



Assuming that u(t) is kept constant along the prediction horizon, that is $N_u = 1$, and with initial condition $y_p(t)$:

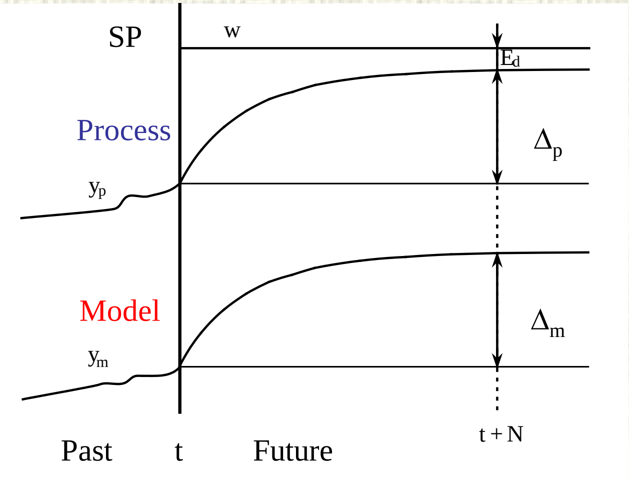
$$y_m(t + N\,T_s) = e^{-\frac{N\,T_s}{\tau}}\,y_p(t) + k\,u(t)\left(1 - e^{-\frac{N\,T_s}{\tau}}\right)$$

T_s sampling time

N prediction (o coincidence) horizon

$$\Delta y_m = \left(e^{-\frac{N\,T_s}{\tau}} - 1\right) y_p(t) - k\,u(t)\left(e^{-\frac{N\,T_s}{\tau}} - 1\right)$$

Design equation



$$\Delta_p = (1 - \lambda^N)(w - y_p(t))$$

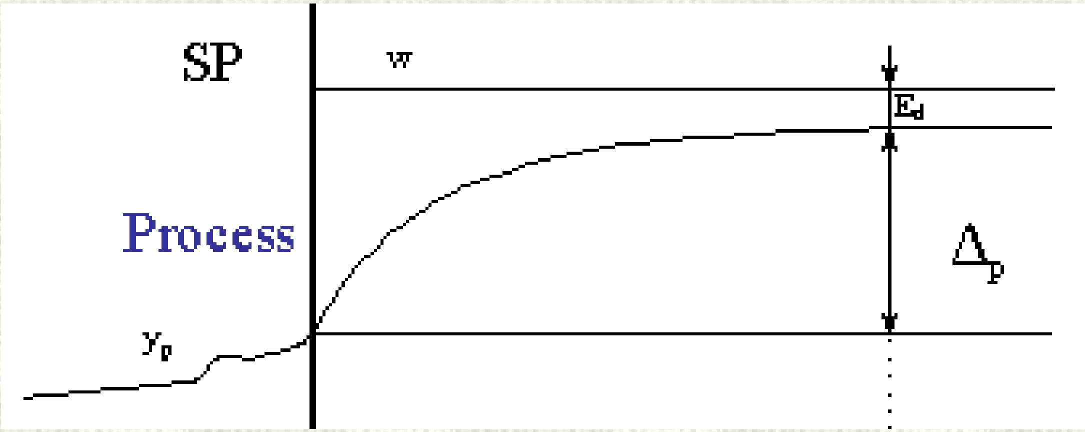
$$\Delta y_m = \left(e^{-\frac{N T_s}{\tau}} - 1\right) y_p(t) - k\, u(t) \left(e^{-\frac{N T_s}{\tau}} - 1\right)$$

design equation: $\Delta_m(u) = \Delta_p$

Explicit solution for the control signal

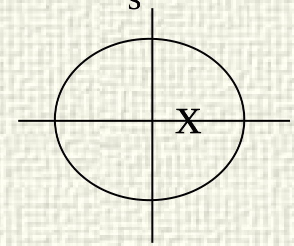
$$u(t) = \frac{\left(e^{-\frac{N T_s}{\tau}} - 1\right) y_p(t) - \left(1 - \lambda^N\right)\left[w(t) - y_p(t)\right]}{K\left(e^{-\frac{N T_s}{\tau}} - 1\right)}$$

Tuning parameters



A discrete first order system with pole λ will give a free response as the one desired at $t+NT_s$

$$\frac{1}{1-\lambda q^{-1}}$$

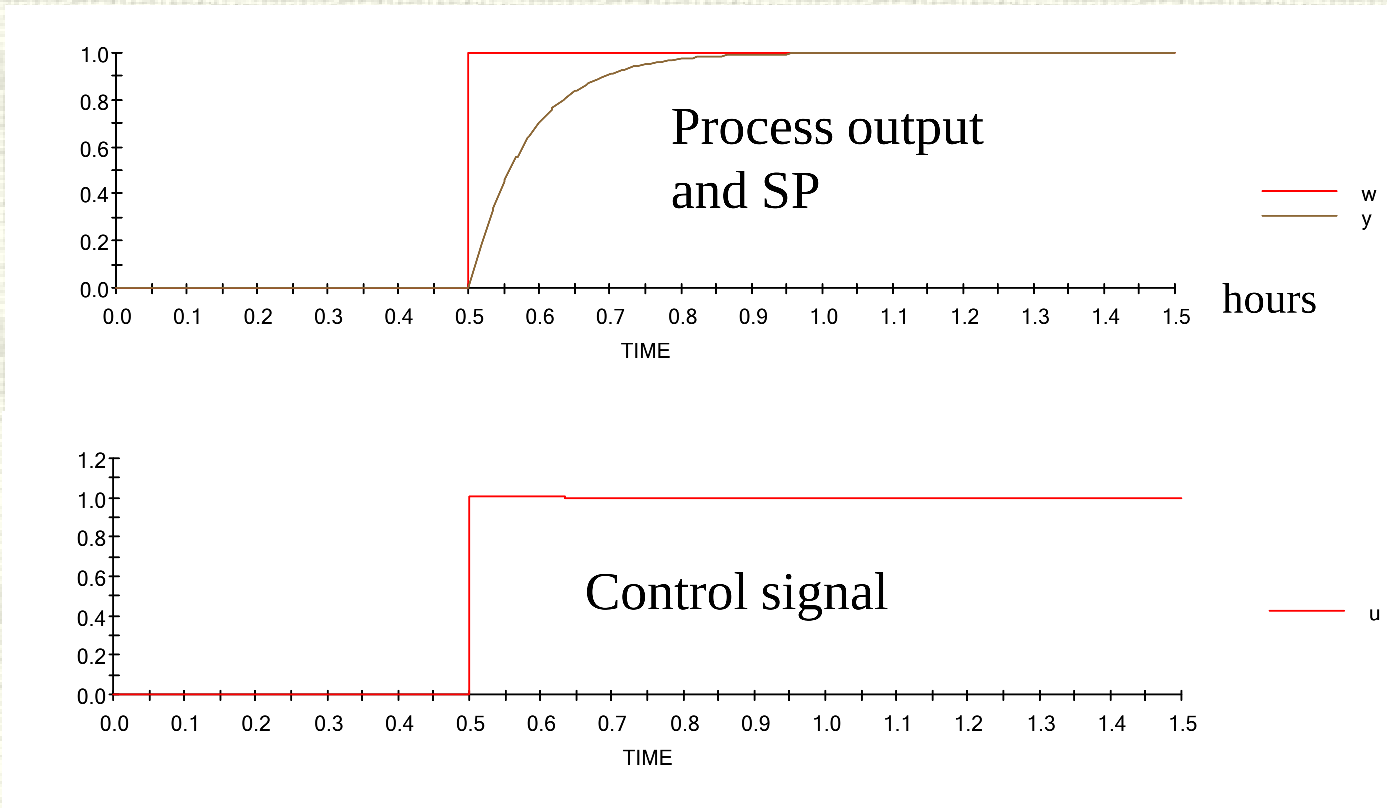


$$u(t)=\frac{\left(e^{-\frac{NT_s}{\tau}}-1\right)y_p(t)-\left(1-\lambda^N\right)\left[w(t)-y_p(t)\right]}{K\left(e^{-\frac{NT_s}{\tau}}-1\right)}$$

N prediction horizon (number of sampling periods required to decrease the current error by λ^N)

λ Reduction factor (0,..,1)

Example. Ideal case



Model = Process

$$G(s) = \frac{1}{5s+1}$$

$K = 1$

$\tau = 5$ min.

$N = 20$

$\lambda = 0.8$

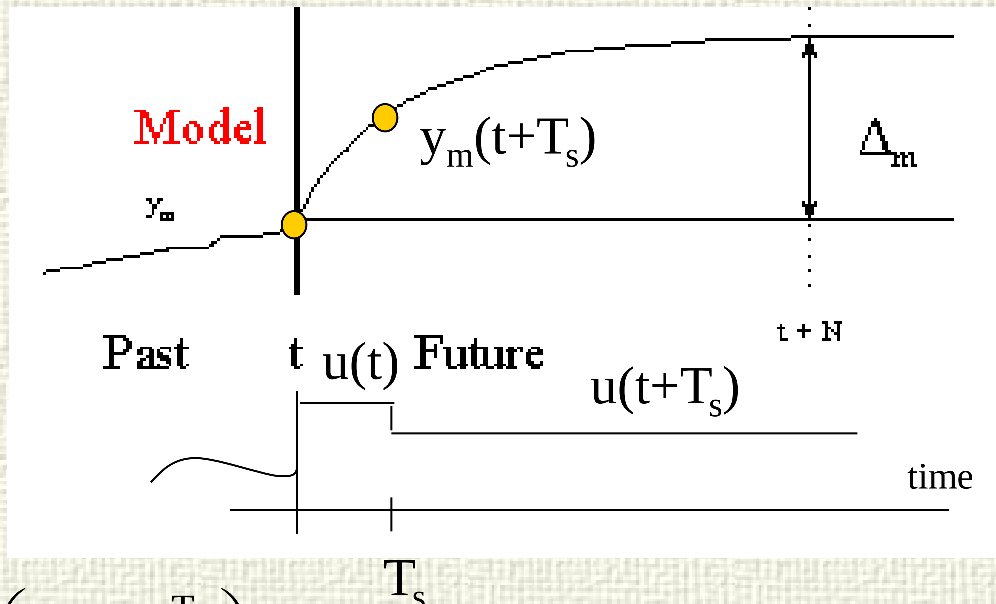
Extension to $N_u > 1$

Example: First order model

$$\tau \frac{dy(t)}{dt} + y(t) = k\,u(t)$$

$N_u = 2$

Predictions are more complex

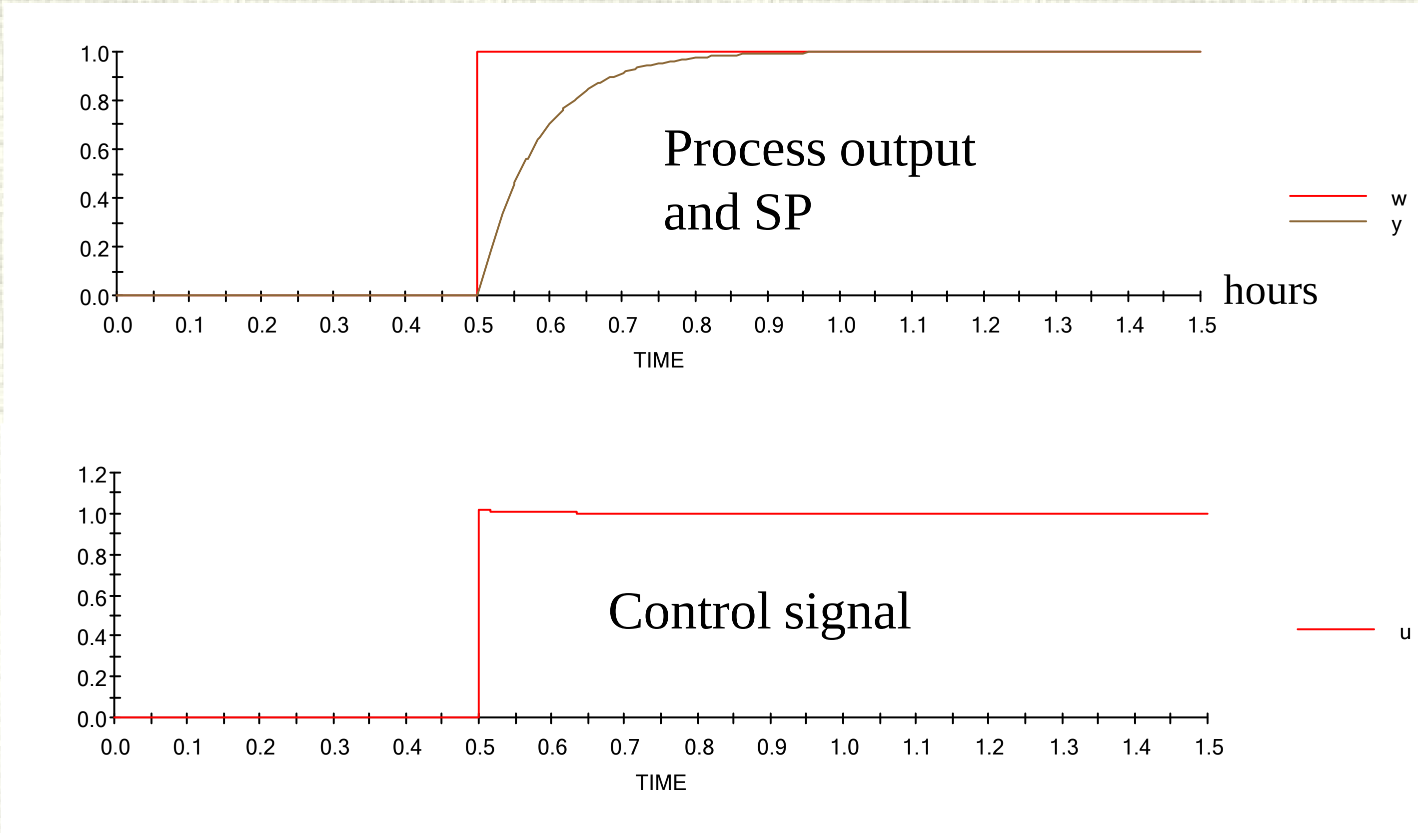


$$y_m(t+T_s) = e^{-\frac{T_s}{\tau}}\, y_p(t) + k\,u(t)\left(1 - e^{-\frac{T_s}{\tau}}\right)$$

Two unknows: u(t), $u(t+T_s)$, then two coincidence points are required

$$y_m(t+N\,T_s) = e^{-\frac{(N-1)\,T_s}{\tau}}\, y_m(t+T_s) + k\,u(t+T_s)\left(1 - e^{-\frac{(N-1)\,T_s}{\tau}}\right)$$

Robustness



10% change in the parameters

Model $\neq$ Process

Model:

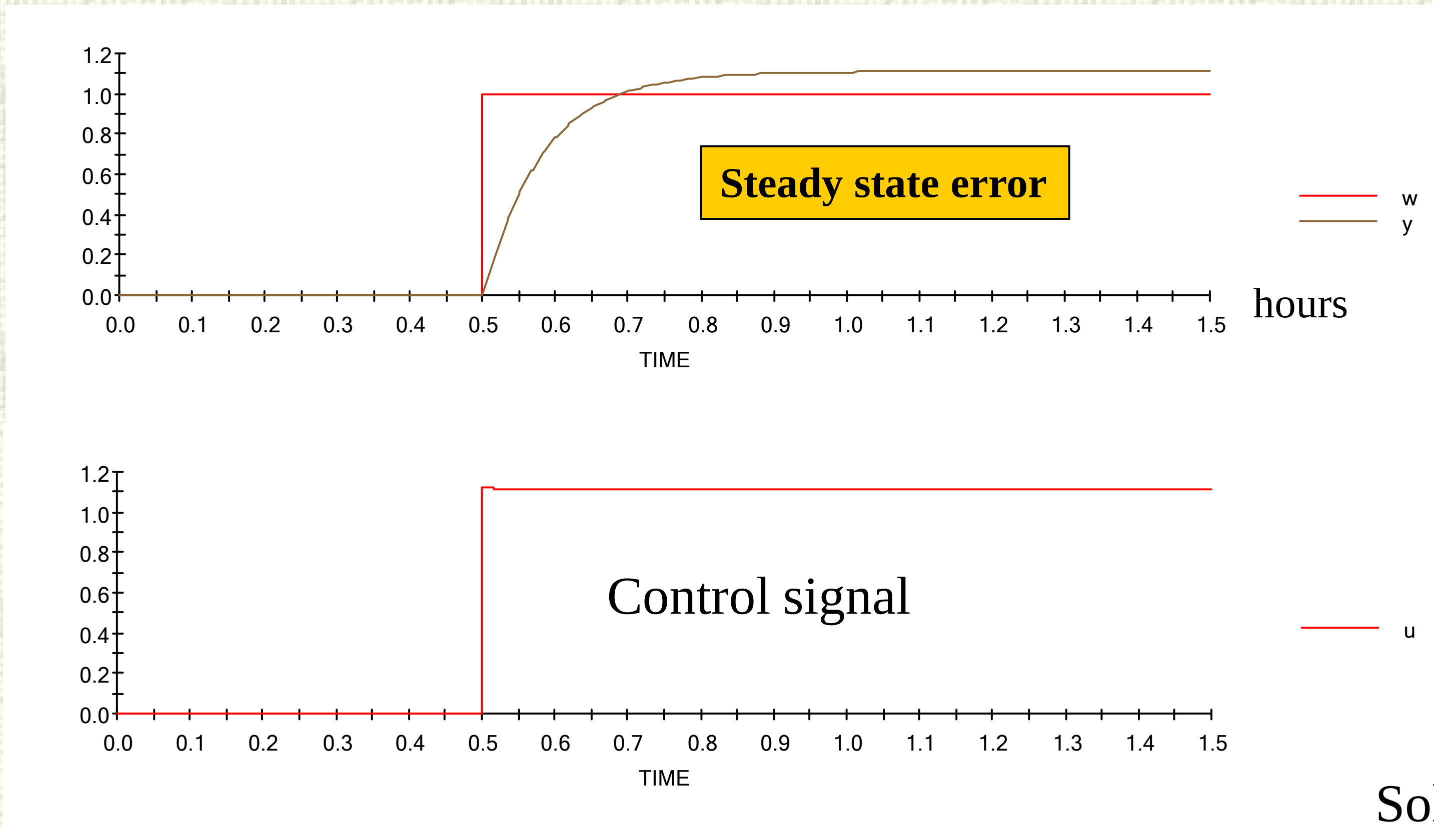
$$G(s) = \frac{1}{5.5s + 1}$$

$N = 20$

$\lambda = 0.8$

A bit slower

Robustness



Model $\neq$ Process

Model:

$$G(s) = \frac{0.9}{5s+1}$$

N = 20

$\lambda = 0.8$

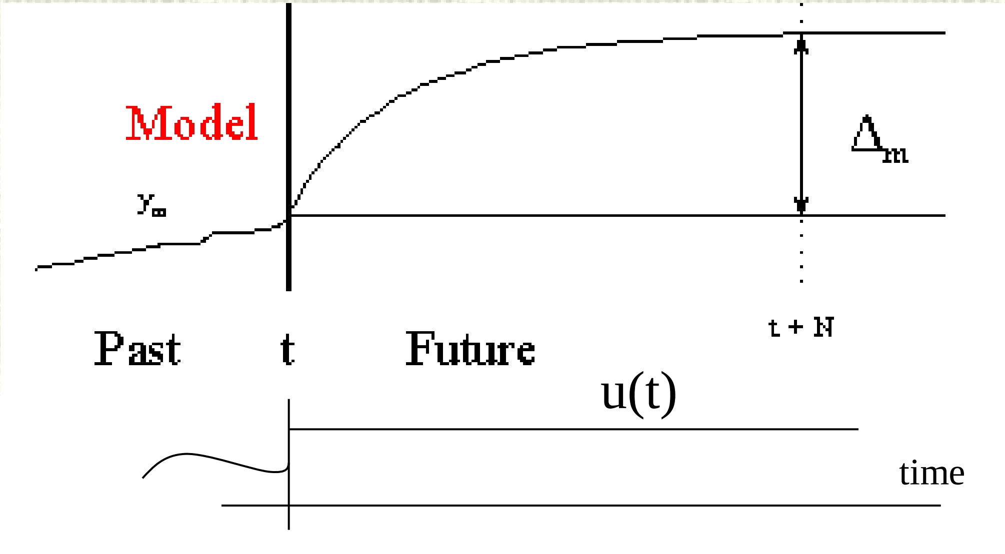
Solution: error model or explicit integrator added

Incorporating errors

Model:

$$\tau \frac{dy(t)}{dt} + y(t) = k\,u(t) + v$$

v disturbance

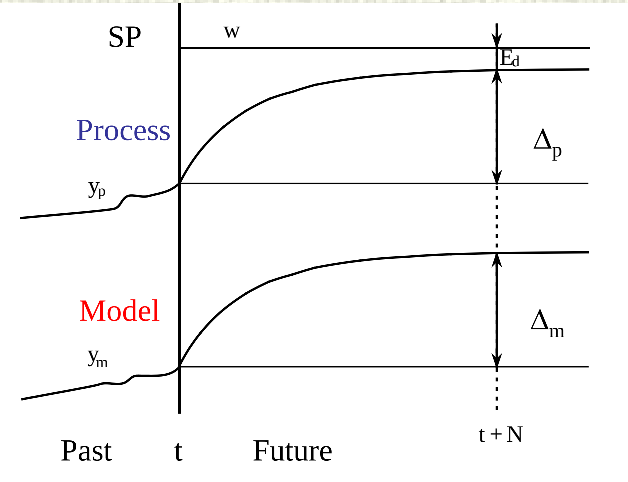


Assuming also that v does not change along the prediction horizon:

$$y(t+NT_s) = e^{-\frac{NT_s}{\tau}} y_p(t) + K\,u(t)\left(1-e^{-\frac{NT_s}{\tau}}\right) + v\left(1-e^{-\frac{NT_s}{\tau}}\right)$$

$$\Delta y_m = \left(e^{-\frac{NT_s}{\tau}} - 1\right) y_p(t) + K\,u(t)\left(1-e^{-\frac{NT_s}{\tau}}\right) + v\left(1-e^{-\frac{NT_s}{\tau}}\right)$$

Design equation



$$\Delta_p = (1-\lambda^N)(w - y_p(t))$$

$$\Delta y_m = \left(e^{-\frac{NT_s}{\tau}} - 1\right) y_p(t) + K u(t)\left(1 - e^{-\frac{NT_s}{\tau}}\right) + v\left(1 - e^{-\frac{NT_s}{\tau}}\right)$$

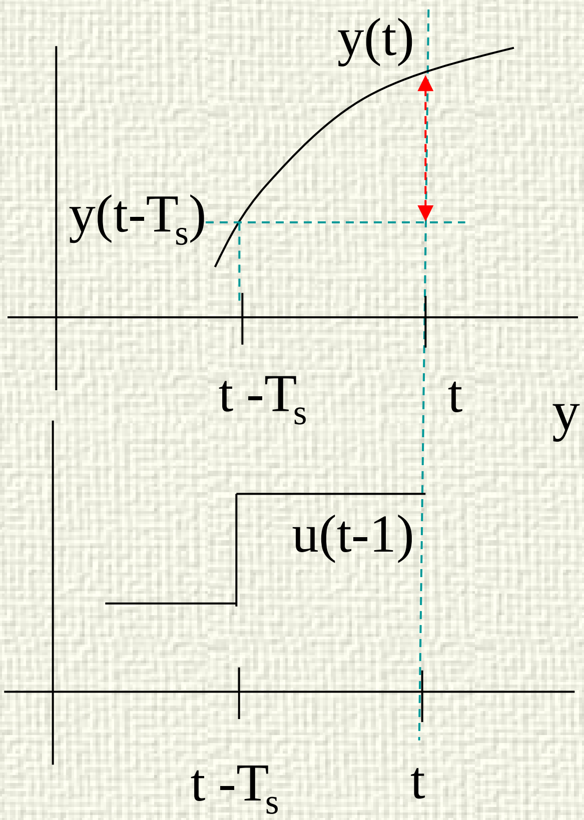
design equation: $\Delta_m(u) = \Delta_p$

Controller equation.

v is not known and needs to be estimated every sampling time

$$u(t) = \frac{\left(e^{-\frac{NT_s}{\tau}} - 1\right)\left(y_p(t) - v\right) - \left(1 - \lambda^N\right)\left[w(k) - y_p(t)\right]}{K\left(e^{-\frac{NT_s}{\tau}} - 1\right)}$$

Estimating v

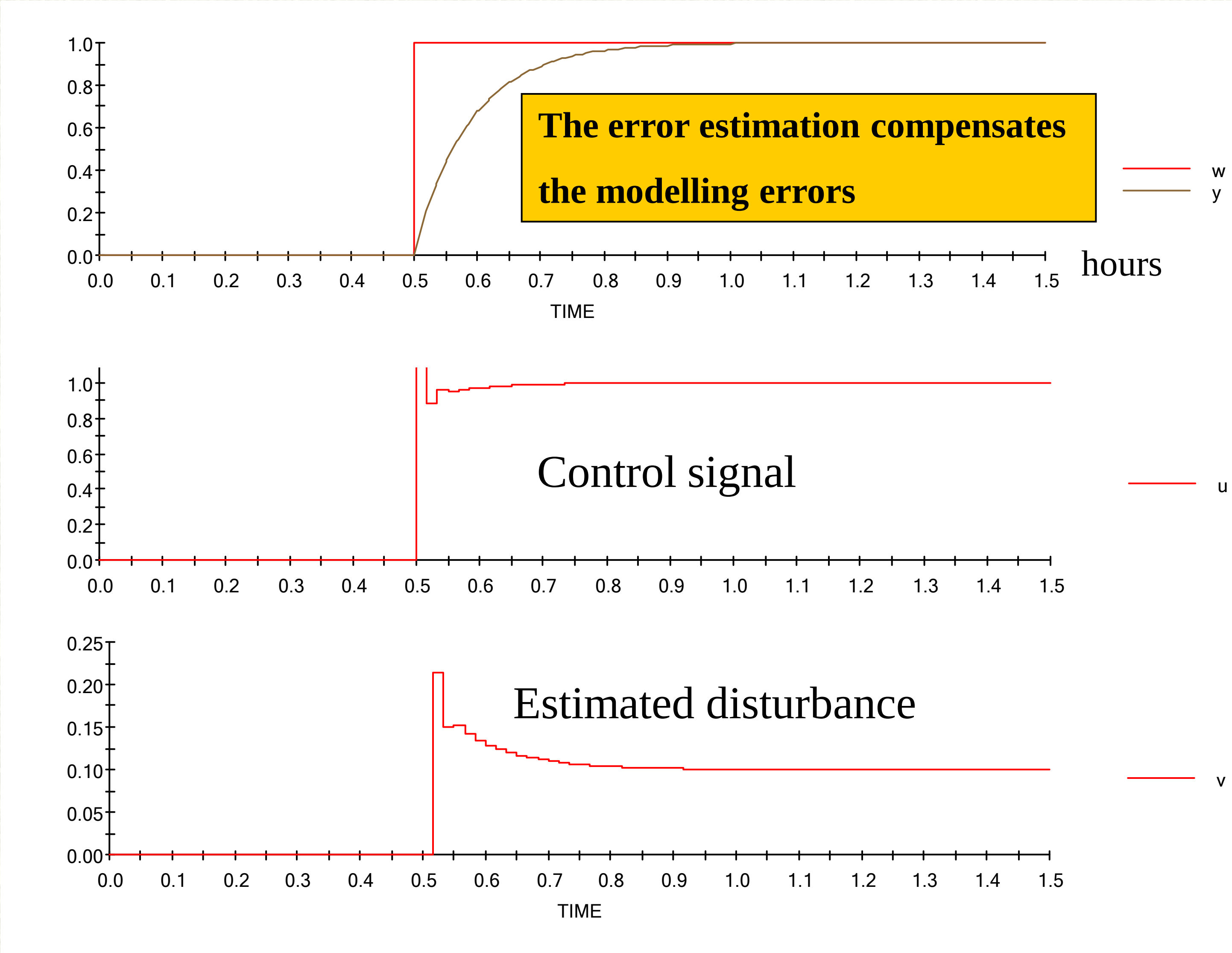


v is estimated from the process model in order to cancel the difference between the measured process output at t and the prediction made with values at t - T_s

$$y_p(t) = e^{-\frac{T_s}{\tau}} y_p(t-T_s) + K u(t-T_s)\left(1-e^{-\frac{T_s}{\tau}}\right) + v\left(1-e^{-\frac{T_s}{\tau}}\right)$$

$$\hat{v} = \frac{y_p(t) - e^{-\frac{T_s}{\tau}} y_p(t-T_s) - K u(t-T_s)\left(1-e^{-\frac{T_s}{\tau}}\right)}{\left(1-e^{-\frac{T_s}{\tau}}\right)}$$

Example



Model ≠ Process

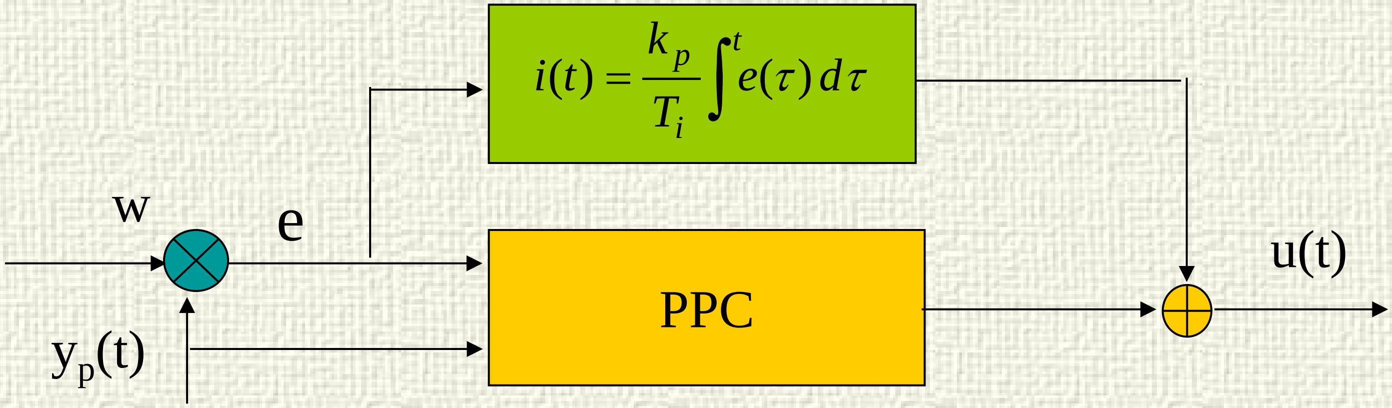
Model:

$$G(s) = \frac{0.9}{5.5s + 1}$$

N = 20

$\lambda = 0.8$

Adding an integrator



$$i(t) = i(t - T_s) + \frac{T_s}{T_i} e(t - T_s) = i(t - T_s) + \frac{T_s}{T_i}(w(t - T_s) - y_p(t - T_s))$$

$$u(t) = \frac{\left(e^{-\frac{N T_s}{\tau}} - 1\right) y_p(t) - \left(1 - \lambda^N\right)\left[w(t) - y_p(t)\right]}{K\left(e^{-\frac{N T_s}{\tau}} - 1\right)} + i\left(t - T_s\right) + \frac{T_s}{T_i} e\left(t - T_s\right)$$

Example

Model ≠ Process

Model:

hours

$$G(s) = \frac{0.9}{5.5s + 1}$$

Control signal

$N = 20$

$\lambda = 0.8$

$T_i = 0.1$ min

$T_s = 20$ sec.

First order + delay

Model:

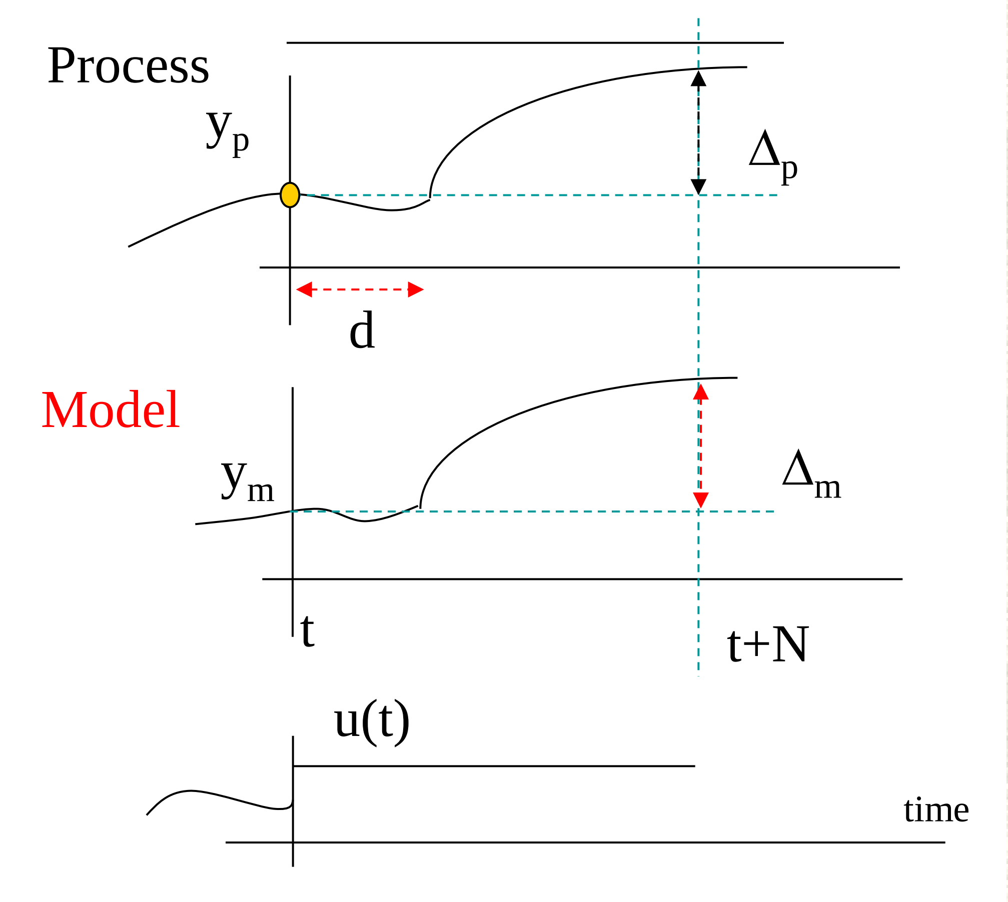
$$\tau \frac{dy(t)}{dt} + y(t) = k\,u(t-d) + v$$

v disturbance

d delay

$d = DT_s$

$$\Delta_p = \left(1-\lambda^{N-D}\right)\left[w(t) - y_p(t)\right]$$



First order + delay

$$\Delta_p = \left(1-\lambda^{N-D}\right)\left[w(t)-y_p(t)\right]$$

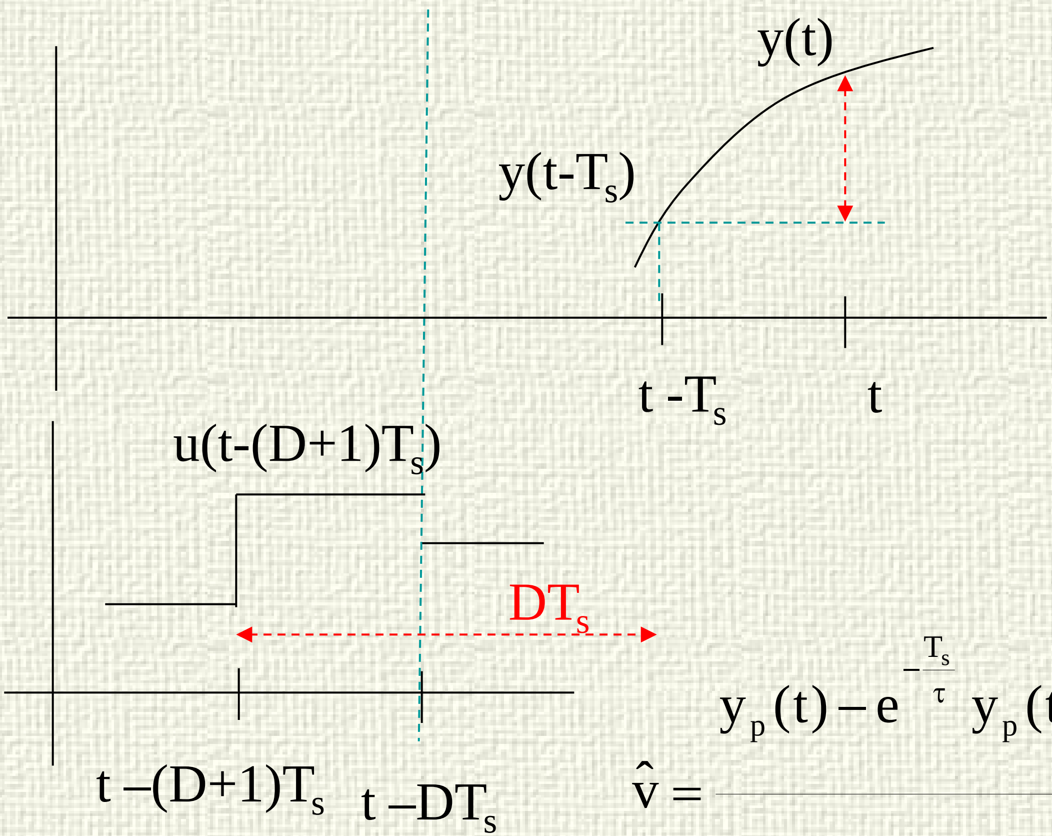
$$\Delta y_m = \left(e^{-\frac{NT_s}{\tau}}-1\right)y_p(t)+e^{-\frac{NT_s}{\tau}}K\left(e^{\frac{T_s}{\tau}}-1\right)\sum_{m=t}^{t+D-1}u(m-D)e^{\frac{(m-t)T_s}{\tau}}+$$

$$+u(t)K\left(1-e^{-\frac{(N-D)T_s}{\tau}}\right)+v\left(1-e^{-\frac{NT_s}{\tau}}\right) \qquad N>D$$

$$u(t)=\left[K\left(1-e^{-\frac{(N-D)T_s}{\tau}}\right)\right]^{-1}\left\{\left(1-\lambda^{N-D}\right)\left[w(t)-y_p(t)\right]-\left(e^{-\frac{NT_s}{\tau}}-1\right)y_p(t)-\right.$$

$$\left.-e^{-\frac{NT_s}{\tau}}K\left(e^{\frac{T_s}{\tau}}-1\right)\sum_{m=t}^{t+D-1}u(m-D)e^{\frac{(m-t)T_s}{\tau}}-v\left(1-e^{-\frac{NT_s}{\tau}}\right)\right\}$$

Estimating v

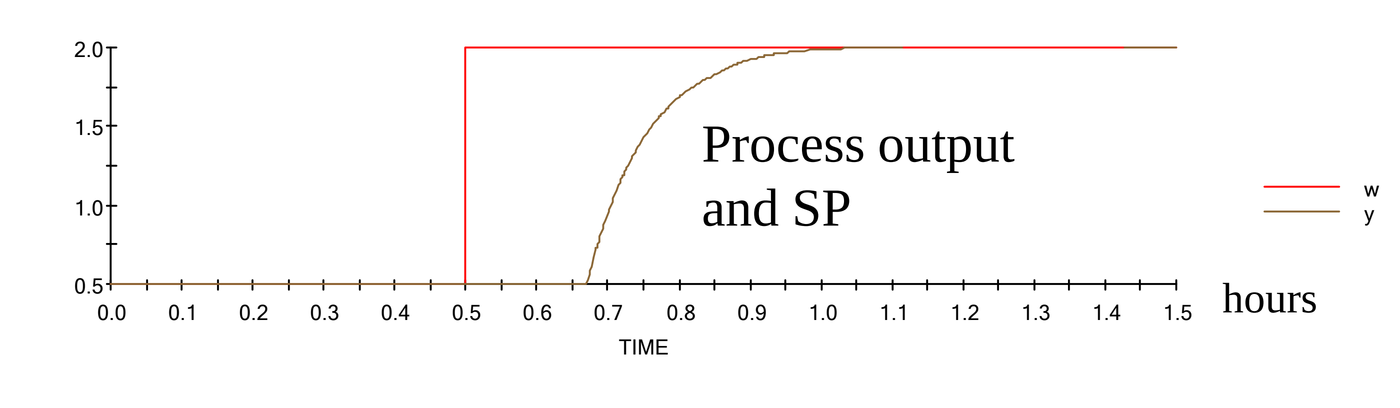


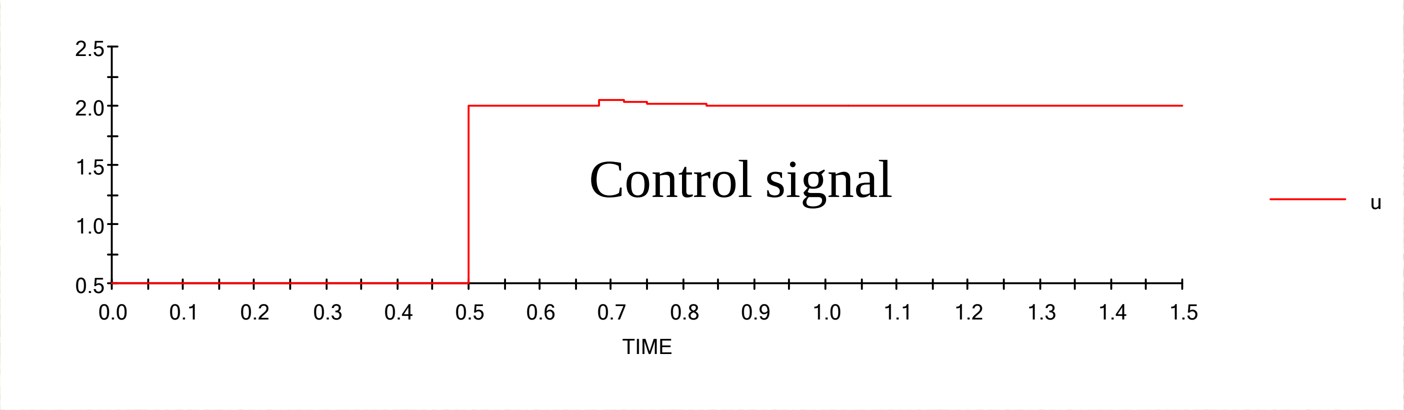
v is estimated from the process model in order to cancel the difference between the measured process output at t and the prediction made with values at t - T_s

$$\hat{v} = \frac{y_p(t) - e^{-\frac{T_s}{\tau}} y_p(t-T_s) - K\,u(t-(D+1)T_s)\left(1-e^{-\frac{T_s}{\tau}}\right)}{\left(1-e^{-\frac{T_s}{\tau}}\right)}$$

+ smoothing filter

Example. Ideal case





Model = Process

$$G(s) = \frac{e^{-10s}}{5s+1}$$

$K = 1$

$\tau = 5$ min.

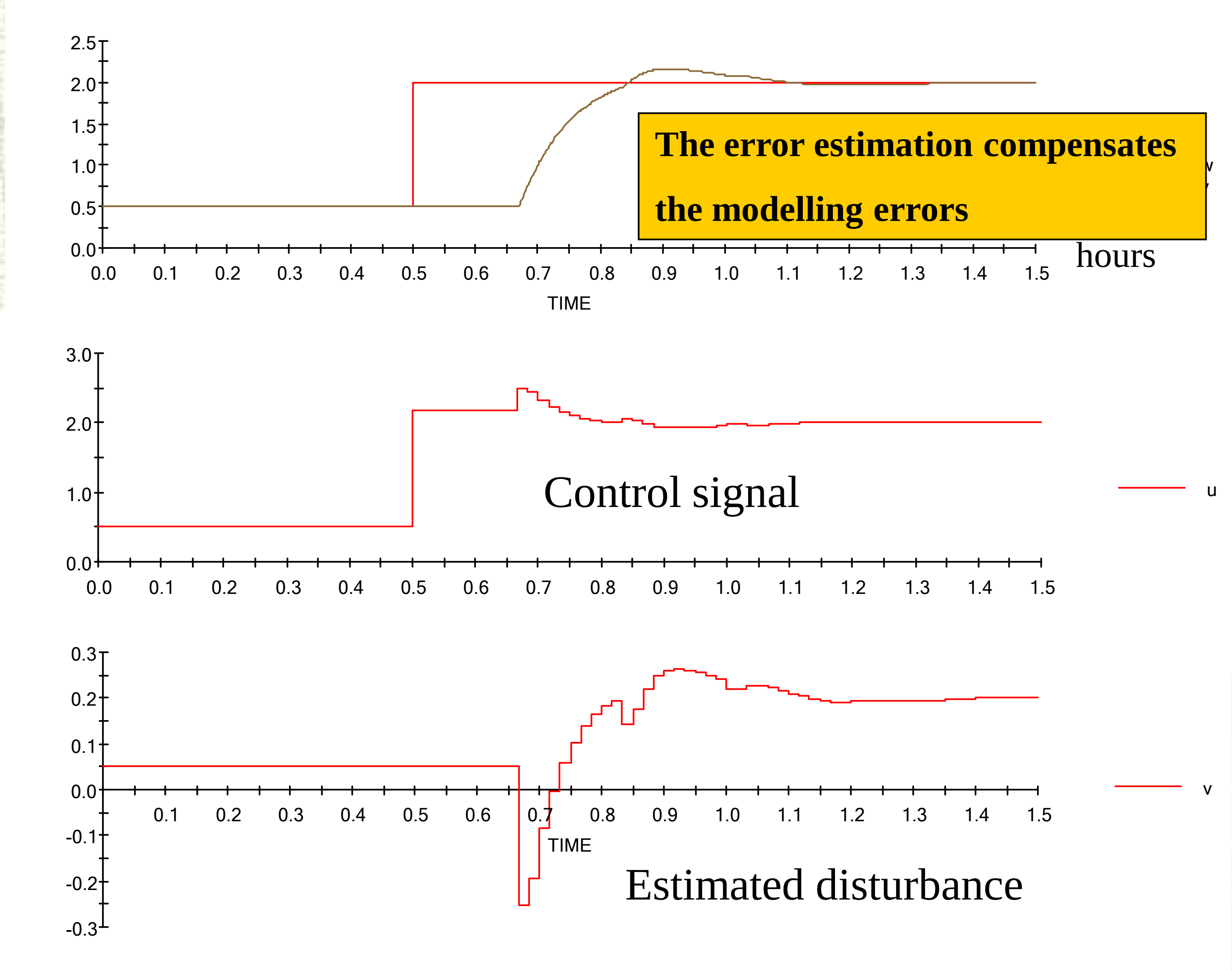
$d = 10$ min.

$N = 40$

$\lambda = 0.8$

$T_s = 1$ min.

Robustness



Model $\neq$ Process

Model:

$$G(s) = \frac{0.9e^{-9s}}{5.5s+1}$$

N = 40

$\lambda = 0.8$